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# Second-order Prejudice Matters \*

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## Abstract

Weak prejudice founds market-wide discrimination. When an employer put the economic efficiency on priority over his/her prejudice, the prejudice rather remains since it incurs no economic disadvantage on him/her. In an equilibrium, preferential treatment toward one category of workers in hiring drives other firms toward competition for that category of workers. While many of the discriminated employees earn low income, however, their employment rate is rather high and a few of them may become superheroes, who earn the highest income of all.

*JEL Classification Number* : D23, J15, J64, J71, L25

*Key words* : second-order prejudice, hiring, discrimination, manpower

## 1. Introduction

Oftentimes, the economic theorists assert that the pressure of economic competition expels prejudiced employers from the market. The storyline is as follows; since the prejudiced employers conduct economically inefficient hiring by rejecting their detested applicants, with free entry, they lose in the competition and are obliged to exit from the market. In this context, they presume some inefficiency or irrationality of the prejudiced employer.

However, the prejudice remains and may matter in the market if the holder of prejudice puts economic efficiency on priority over his/her prejudice. First, he/she survives the pressure of free entry because he/she maximizes his/her monetary payoff. Second, if the market holds some kind of friction, the prejudiced person may have opportunities and power to satisfy his/her prejudice.

The aggregate effect of this second-order prejudice is the theme of this paper. This study is, in a sense, a microfoundation of prejudice-based labor market discrimination. It is based on a model wherein each firm with a limit of manpower for interviewing matches with multiple applicants and sometimes screens them to fit the scale of manpower. Mainly, the study shows three points. First, discriminatory screening by the firms makes majority of the discriminated workers lower income earners. In a case, even those firms without prejudice follow the behavior of the prejudiced firms. Second, however, the employment rate of the discriminated workers is not necessarily lower than the others. Further, a small part of them might be the highest income earner. Third, this discrimination has a robustness against the pressure of free entry.

Classically, Becker (1953) formulated the purification effect against discrimination of the free entry. Several studies on discrimination followed it; for example, the statistical discrimination (Arrow 1973, Coate and Loury 1993) and discrimination in the search market (Black 1995). This paper is especially related to Black (1995), who formulated the externality of discrimination; when some prejudiced firms discriminate against some kind of workers, such workers' employment rate decreases, so that even the unprejudiced firms offer lower wage for the discriminated workers. However, even in the search market, the free entry pressure has expelled the prejudiced employers, since they spoil fruitful hiring opportunities. This paper, on the other hand, investigates a style of prejudice that affects the market *and* survives the competition with free entry.

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The rest of the paper has the following construction. Section 2 introduces the mathematical model of labor market. Section 3 analyzes the model and shows the main results. Section 4 briefly makes practical discussions on the results. Section 5 concludes the paper.

## 2. Model

The model is dynamic with three stages. At the stage 1, the round 1 of random matching labor search market opens. At the stage 2, the round 2 of the same labor market opens. At the stage 3, each firm and its hired workers co-work to produce monetary payoff. Both workers and firms are assumed to be perfectly patient.

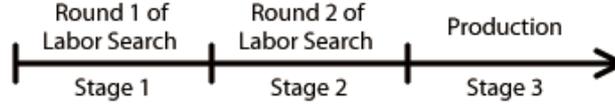


Figure 1: Timings

Workers are continuously infinite with size of  $L$ . They have two types  $a$  and  $b$ ; the size  $A$  ( $< L$ ) of them are of type  $a$  and the rest  $B$  ( $\equiv L - A$ ) of them are of type  $b$ . The types are costlessly observable and do not affect the productivity of workers. Proportion  $p$  ( $< 1$ ) of the workers are potentially qualified. Neither workers nor firms know who of the workers are potentially qualified.<sup>1</sup> Each worker maximizes his/her expected wage level.

There are also (uniformly distributed) continuously infinite firms with size of  $J$ . Each firm (in  $(0, J)$ ) holds  $m$  of manpower. It implies that a firm can interview at most the density  $m$  of workers in one round of the labor search. In an interview with a potentially qualified worker, a firm finds with probability of  $q$  ( $< 1$ ) that the worker is (practically) qualified.<sup>2</sup> If the interviewed worker is not potentially qualified, he/she is never found to be qualified. A firm can hire only qualified workers. If a firm hires a qualified worker, they produce monetary payoff of  $v$  ( $> 0$ ) at the stage 3. Not depending on how many workers a firm hires, it owes a fixed capital cost  $\kappa$ .

The labor market is a random matching search market. In each round of the labor search, each firm matches with the density  $L(t)/J$  of workers.  $L(t)$  is the size of unemployed workers at stage  $t$  ( $\in \{1, 2\}$ ), with  $L(1) = L$ . Let  $A(t)$  and  $B(t)$  be the size of type  $a$  and  $b$  unemployed workers at stage  $t$ , respectively. Among the density  $L(t)/J$  of workers whom a firm matches with,  $A(t)/J$  are of type  $a$  and  $B(t)/J$  are of type  $b$ . Among those applicants, each firm interviews the density  $\min\{m, L(t)/J\}$  of workers.<sup>3</sup>

If  $L(t)/J > m$ , each firm selects its screening policy  $s(t)$  from  $\{a, b, r\}$ .  $a$  implies priority for type  $a$ ; by selecting  $a$ , the firm interviews the density  $\min\{A(t)/J, m\}$  of type  $a$  workers and the density  $\max\{0, m - A(t)/J\}$  of type  $b$  workers. By selecting  $b$ , on the other hand, the firm interviews the density  $\max\{0, m - B(t)/J\}$  of type  $a$  workers and the density  $\min\{B(t)/J, m\}$  of type  $b$  workers. When it selects  $r$ , random policy, it interviews the density  $\min\{\frac{A(t)m}{A(t)+B(t)}, A(t)/J\}$  of type  $a$  workers and the density  $\min\{\frac{B(t)m}{A(t)+B(t)}, B(t)/J\}$  of type  $b$  workers. Those workers who are not interviewed just miss their job opportunities at the current stage.

With each worker who is found to be qualified, the Nash bargaining process between the firm and the worker provides a wage (and job) offer. The bargaining power of worker is  $\alpha \in (0, 1)$ . Let  $R(t)$  be the reservation wage at stage  $t$ : that is, the expected wage that the worker gains if he/she rejects this job offer. The wage offer  $w(t)$  is  $w(t) = R(t) + \alpha(v - R(t))$ . It is assumed that

<sup>1</sup>This setting ensures that all the unemployed workers visit the second round of labor market.

<sup>2</sup>Each worker who are found to be qualified comes to recognize that he/she has been potentially qualified.

<sup>3</sup>Due to the person-wise production technology and the infinity of firms, it is not restrictive to assume that each firm interviews and hires as many workers as possible.

the worker accepts the offer whenever  $w(t) > R(t)$  so that he/she always accepts it.<sup>4</sup> In the case he/she rejects it, it is assumed that his/her qualification is neither observable nor verifiable.

At the stage 3, the production realizes. In each employment contract signed at stage  $t$ , the worker gains monetary payoff of  $w(t)$  and the firm gains monetary payoff of  $v - w(t)$ . The firm's total monetary payoff is derived by the integration on its employees.

Each firm has its preference over profiles of monetary payoff and terms of hiring. Simply, let  $R_+$  be the space of monetary payoff. Let  $h(t) \equiv \{l_a(t), l_b(t), w_a(t), w_b(t)\}$  be the terms of hiring at stage  $t$ .  $l_k(t)$  is the density of type  $k$  workers whom the firm hires at stage  $t$ .  $w_k(t)$  is the wage that the hired type  $k$  worker receives; by the setting,  $w_k(t)$  is the same for the same type of workers at the same stage. Each firm's preference is defined over  $R_+ \times \{h(1)\} \times \{h(2)\}$ . As the common feature, each firm takes first priority over monetary payoff in the lexicographic manner. That is, for an arbitrary pair of  $(Y, h(1), h(2))$  and  $(y, h'(1), h'(2))$ , if  $Y > y$ ,  $(Y, h(1), h(2)) \succ (y, h'(1), h'(2))$ . The level of monetary payoff is, specifically,  $\sum_{t=1,2} \sum_{k \in \{a,b\}} (v - w_k(t)) l_k(t)$ .

The size  $D$  of firms are *second-order prejudiced* for type  $a$ . A second-order utility function  $u : \{h(1)\} \times \{h(2)\} \rightarrow R_+$  expresses the prejudice. Let us define  $t^*$  and  $t'$  as  $t^* \equiv \arg \max_{t \in \{1,2\}} w_a(t)$  and  $t' \equiv \arg \min_{t \in \{1,2\}} w_b(t)$ , respectively. Then,  $u(h(1), h(2)) \equiv (w_a(t^*) - w_b(t'))(l_a(t^*) - l_b(t^*))$ . For an arbitrary pair of  $(Y, h(1), h(2))$  and  $(y, h'(1), h'(2))$ ,  $(Y, h(1), h(2)) \succeq (y, h'(1), h'(2))$  iff  $u(h(1), h(2)) \geq u(h'(1), h'(2))$ ; the preference relation is strict if and only if the inequality is strict. If either  $t^*$  or  $t'$  do not exist,  $w_a(t^*) - w_b(t')$  is set to be zero. It is assumed that the prejudiced firm conducts the screening policy  $a$  if  $w_a(t^*) - w_b(t') = 0$  and the screening does not change its monetary payoff. The rest of the firms, on the other hand, are "unprejudiced" in the sense that they are egalitarian. Their preference over the policies is represented by  $r \succ a \sim b$ , which they follow whenever the policies are monetarily irrelevant.

The sequential rounds of labor market intuitively correspond to a substantial length of firms' recruiting process with some threshold timing with which the terms of trade change. In the academic job market of the US, for example, the American Economic Association conference in January is a typical threshold. In Japan, undergraduates have about one and a half years of term for job research, from the autumn of the third-year; July of the fourth-year is a typical threshold.

The formulation of prejudice indicates two characteristics of the prejudiced firm. First, it has no intention to decrease its profit in order to satisfy its prejudice. Second, its prejudice is more satisfied when higher wage group includes more type  $a$  workers than type  $b$  counterparts.

### 3. Analysis

Straightforwardly, no discrimination (i.e. screening) occurs when  $m \geq L/J$ . Therefore, henceforth the author focuses on the cases wherein  $L/J > m$ . Throughout the analysis, the equilibrium notion is the subgame perfect equilibrium. Further,  $R(2) = 0$  is granted by the setting. Based on this fact, the author shows the nondecreasingness of the (reservation) wage.

#### Lemma 1

*For each worker,  $R(1) \geq R(2)$  ( $= 0$ ). This implies  $w(1) \geq w(2) = \alpha v$ . The inequality is strict when the worker has positive probability of being hired at stage 2.*

**Proof:** See A1 of the Appendix.

The intuition is plausible; since there is no job opportunity after stage 2, each worker's reservation wage at stage 2 is 0. Positive expected wage at stage 2 makes the terms of trade at stage 1 better for the workers. Then, the prejudiced firms tend to conduct the discriminatory screening. The lemma below shows a rather moderate case wherein the discrimination has no externality.

<sup>4</sup>The results are robust even if you consider the possibility of on-the-job search. See Lemma 1.

**Lemma 2**

Assume  $L/J > m \geq L/J - mpq$ . The prejudiced firms conduct screening policy *a* in stage 1. The unprejudiced firms, on the other hand, conduct *r*.

**Proof:** See A2 of the Appendix.

The condition in the statement expresses the situation wherein each firm has the opportunity of screening only at stage 1. At stage 2, every firm interviews all the workers whom it matches with. Then, the screening at stage 1 do not change the firm's monetary payoff. The prejudiced firms take advantage of this opportunity to satisfy their prejudice.

As the readers may notice, the screening imposes disparate impacts on type *a* and *b* workers.

**Proposition 1**

Assume  $L/J > m \geq L/J - mpq$ . For each worker,  $w(1) = \alpha v\{1 + q(1 - \alpha)\} > w(2) = \alpha v$  stands. *Ex ante*, type *a* (resp. *b*) workers have higher probability of earning  $w(1)$  (resp.  $w(2)$ ) than workers of another types.

**Proof:** See A3 of the Appendix.

Type *a* workers tend to earn higher wage. This productivity-irrelevant discrimination occurs due to the friction (i.e. manpower limit) in interviewing process and the firms' power of screening. The prejudiced firms enjoy the room for screening in preferable way for their favorite type.

In the case above, type *a* workers are better off also with respect to the employment rate.

**Lemma 3**

Let  $L_a^*$  and  $L_b^*$  be the total size of hired type *a* and *b* workers, respectively. If  $L/J > m \geq L/J - mpq$ ,  $L_a^*/A > L_b^*/B$  stands.

**Proof:** See A4 of the Appendix.

Type *a* workers are treated warmly at stage 1, and equally with type *b* workers at stage 2. Naturally, their employment rate becomes higher than that of type *b* workers.

**Discrimination Externality and Reactionary Screening**

If the manpower limit matters at each round of labor market, the discrimination generates externality so that all the firms follow one manner of discrimination. The focus here is on the cases wherein  $L/J - mpq > m$ . Under this condition, two sequential screenings occur; the total amount of unemployed workers is always too large for the firms' total amount of manpower.

When a nonnegligible size of firms discriminate for type *a* workers at stage 1, each firm has an incentive to tap the leftovers: that is, type *b* workers. This is because the first screening makes type *a* workers as a group qualitatively inferior to type *b* workers with respect to the residual proportion of potentially qualified workers; there remains relatively small percentage of potentially qualified workers among type *a* workers, on comparison with type *b* workers, at stage 2. Then, each firm competes for type *b* workers at stage 2 in order to find as many qualified workers as possible.

This reactionary screening at stage 2, in turn, creates an incentive for the firms to discriminate against type *b* workers at stage 1. Due to the reactionary screening, the reservation wage at stage 1 for type *b* worker is higher than that for type *a* worker. It makes  $w_b(1)$  higher than  $w_a(1)$ . Therefore, each firm rationally competes for type *a* workers, in order to utilize cheaper laborforce. The proposition below summarizes this story:

**Proposition 2**

Assume  $L/J - mpq > m$  and  $m > B/J - \max\{0, m - A/J\} \cdot pq \cdot D/J - Bm/(A+B) \cdot pq \cdot (J-D)/J$ . Then, there exists an equilibrium wherein all the firms, whether prejudiced or unprejudiced, take the screening policy *a* and *b* at stages 1 and 2, respectively. The equilibrium outcome satisfies

$w_a(1) > w_a(2) = w_b(2) = \alpha v$ . For each firm, if  $m > A/J$ ,  $w_b(1)$  exists and  $w_b(1) > w_a(1)$ .

**Proof:** See A5 of the Appendix.

Note that the prejudiced firms also take the policy  $b$  at stage 2. Since all the firms respect their monetary payoffs the most of all, at stage 2, the prejudiced firms also input their manpower into the more fertile source of laborforce: that is, type  $b$  applicants. Also note that type  $b$  workers have to be of appropriately small population for the disparate impact to remain. The second condition in the statement indicates this point. Its intuitive meaning is that type  $a$  workers have positive probability of being interviewed at stage 2, which is the precondition for that they are hired with higher wage offer at stage 1.<sup>5</sup>

It might be surprising that high-earning type  $b$  worker earns further higher than high-earning type  $a$  worker. The existence of such a superhero is due to the reactionary screening for type  $b$  workers at stage 2. This treatment creates better outside options for type  $b$  workers at stage 1. Therefore, if he/she is luckily found to be qualified at stage 1, his/her income becomes the highest. This superhero appears when the original size of type  $a$  workers ( $A$ ) is small enough for type  $b$  workers to be hired at stage 1 with positive probability.

Due to the reactionary screening, whether type  $a$  workers face higher employment rate than type  $b$  workers becomes unclear.

#### Lemma 4

*Whether  $L_a^*/A \geq L_b^*/B$  stands or  $L_b^*/B > L_a^*/A$  stands depends on the parameters. For example, there is thresholds  $L_+$  and  $B_+$  such that if  $L_+ > L$  and  $B_+ > B$ ,  $L_a^*/A \geq L_b^*/B$ . On the other hand, there are thresholds  $B_-$  and  $q_-$  such that if  $B > B_-$  and  $q > q_-$ ,  $L_b^*/B > L_a^*/A$ .*

**Proof:** See A6 of the Appendix.

Intuitively, if type  $b$  workers are substantially small minority, the reactionary screening does not work well enough to recover their employment rate. On the other hand, if they can be major applicants at the second round of job search and the interviewers' evaluating abilities are reliable, they may be better off than type  $a$  counterparts with respect to the employment rate. On comparison with Lemma 3, the reactionary screening has a nonnegligible power to offer job opportunities for the discriminated type  $b$  workers.

**Remark:** As a natural corollary of the proposition 2, there also exists an equilibrium wherein all the firms take the policy  $b$  and  $a$  at stages 1 and 2, respectively. However, there cannot be an equilibrium wherein all the firms conduct  $r$  at every stage; because, with such a strategy profile given, the prejudiced firms voluntarily deviate to conduct  $a$  at every stage.

#### Prejudice Backrush Test

Given the multiplicity of equilibria, in this subsection, the author considers the equilibria's evolutionary robustness against a perturbation on the firms' strategies. Let us assume that the two equilibria with discrimination externality coexist, with  $m > A(2)/J$  and  $m > B(2)/J$ . Suppose that the proportion  $e$  ( $\in (0, 1)$ ) of the firms (randomly chosen) mutate so that their prejudice becomes of the first order; that is, the mutated prejudiced and unprejudiced firms, respectively, come to conduct the policies  $(s(1), s(2)) = (a, b)$  and  $(s(1), s(2)) = (r, r)$ .

This mutation intuitively corresponds to a collective backrush based on the prejudice, since it indicates that a part of the firms try to shift the equilibrium from one to another. Given the coexistence of two equilibria with discrimination, the prejudiced firms, particularly, clearly prefer one for type  $a$  workers to another for type  $b$  workers. By collectively deviating from the equilibrium

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<sup>5</sup>Even if this second condition is violated, there still exists an equilibrium wherein the firms' policies are the same. However, in that case, all the type  $a$  workers earn the lowest wage.

strategy, the deviating firms might be able to induce other firms to follow their deviation, so that the equilibrium shifts. If the shift occurs, the prejudiced deviators owe relatively no cost since they come to play the equilibrium strategy.

An equilibrium is *robust against the prejudice backrush* if, for each nonmutated firm, its equilibrium strategy maximizes its profit when other nonmutated firms keep playing their equilibrium strategies. With this definition, the author investigates the invasion barriers of the two equilibria with discrimination externality. Let  $(a, b)$  (*resp.*  $(b, a)$ ) be the name of equilibrium with  $(s(1), s(2)) = (a, b)$  (*resp.*  $(s(1), s(2)) = (b, a)$ ). The following lemma shows that the backrush always matters for the equilibrium  $(b, a)$ .

**Lemma 5**

*Suppose that the two equilibria with discrimination externality coexist, with  $m > A(2)/J$  and  $m > B(2)/J$  standing for both. The equilibrium  $(a, b)$  is always robust against the prejudice backrush. The equilibrium  $(b, a)$  is robust against the prejudice backrush if and only if  $\epsilon^* > e$  for a threshold  $\epsilon^* (< 1)$ .  $\epsilon^*$  is a decreasing function of  $D$ .*

**Proof:** See A7 of the Appendix.

Since the invasion of prejudice is intended to realize the equilibrium  $(a, b)$ , it never harms its existence. The mutation of the unprejudiced firms is qualitatively not harmful for both equilibria. The equilibrium  $(b, a)$ , on the other hand, has a degree of vulnerability to the invasion. When the mutation occurs for sufficiently large proportion of the firms, the equilibrium  $(b, a)$  naturally loses its supporters, and therefore, its status of the established institution. Its basin of attraction,  $\epsilon^*$ , is smaller when the size of the prejudiced firms, the potential invaders, is larger.

This instability result implies that the existence of majority prejudice may matter for the selection of equilibria. Although the second-order prejudice does not affect the equilibrium behavior when the discrimination has externality, it still can be the source of incentive for the prejudiced firms to shift the behavior by invading the economy with the prejudice backrush. If it is, there is more likely to realize an institution such that satisfies the prejudice more.

**Free Entry Test**

Finally, the author considers the economy with free entry. Given the capital cost  $\kappa$ , the scale of firms,  $J$ , is determined so that each firm's net profit becomes exactly zero.  $D/J$  does not have to be the same to that in the previous analysis. There are four possible outcomes of this labor market: that is, no discrimination, discrimination without externality, discrimination for type  $a$  workers with externality, and discrimination for type  $b$  workers with externality. The author abbreviates each equilibrium outcome as ND, DOE, DWEa, and DWEb, respectively.

As the readers may expect, the market competition does not expel the prejudiced firms. Every firm, whether prejudiced or unprejudiced, earns the same profit in each equilibrium. Therefore, even if the free entry is assumed, the prejudiced firms just keep running their business. The equilibrium outcome depends on the level of  $\kappa$ .

**Proposition 3**

*For an arbitrary level of  $D/J \in (0, 1)$ , there exists an equilibrium with free entry wherein the ratio of the prejudiced firms to all the firms is  $D/J$ . Whether the equilibrium outcome is ND, DOE, or DWE depends on the level of  $\kappa$ .*

- 1) *If  $mpq(1 - \alpha)(2 - q - \alpha q)v \geq \kappa$ , the outcome is ND.*
- 2) *If  $mpq(1 - \alpha)(2 + pq - q - \alpha q)v \geq \kappa > mpq(1 - \alpha)(2 - q - \alpha q)v$ , the outcome is DOE.*
- 3) *If  $mpq(1 - \alpha)2v > \kappa > mpq(1 - \alpha)(2 + pq - q - \alpha q)v$ , the outcome is DWEa or DWEb.*

**Proof:** See A8 of the Appendix.

The free entry does not necessarily resolve the problem of discrimination. When the firm-wise capital cost is sufficiently high, the outcome DOE or DWE still appears in the equilibrium with free entry. Further, encouraging the firms' entry is sometimes, from a viewpoint, undesirable. In an equilibrium with outcome of DOE, type  $b$  workers suffer from both lower wage and lower employment rate. In an equilibrium with outcome of DWEa, on the other hand, though their majority still earns lower wage, their employment rate is competent with type  $a$  counterparts and a few of them become superheroes. When you attach importance to relative equality among different types of employees, you might conclude that the DWEa is rather better than the DOE.

Let the author note on the total employment rate. If you select the employment rate of total workers as the welfare standard, the equilibrium with ND is undoubtedly the best. In that equilibrium, each worker has two sequential opportunities of interview. Therefore, given the level of  $L$ , the most workers are hired of the four equilibria.

#### 4. Discussions and Related Literatures

Some readers may wonder whether it is legally possible to pay different wages for equally qualified workers. However, finding premises for such a compensation scheme is often possible, even if it is not easy. Particularly, when the timings at which the workers are hired are different, they can be classified into different kinds of contracts: for example, long-term and short-term contracts or tenured and nontenured job offers. And identifying that the difference is due to the prejudice frequently incurs a sizable cost (Epstein 1992).

Since the holder of prejudice is sometimes not identifiable, the regulation on the prejudice is logically difficult. And, whether the unprejudiced members are *responsible* for the discrimination or not is a difficult question by itself; they are just doing their best to survive the competition. Further, the unprejudiced behavior of market is hard to accomplish by the firms' free conducts. As a solution, a regulator might be obliged to implement the public quota on the labor market. On the issue of quota, the public quota is said to be economically more efficient than the firms' implicit (voluntary) quotas driven by an auditor's monitoring (Fryer 2007).

#### 5. Conclusion

This study has shown three points. 1) The prejudice whose priority is secondary to monetary payoff still affects the labor market: particularly, wage dispersion, employment rate dispersion, and/or the selection of market discrimination. 2) While the discriminated workers are likely to earn lower income, however, their employment rate can be competent with other workers. 3) Firms that hold the second-order prejudice remain even if the entry is free. Given these results, identifying and/or expelling the prejudice in labor market seem to be seriously difficult. A policymaker might want to implement the public quota for some cases.

#### Appendix

##### A1. Proof of Lemma 1

$R(2) = 0$  because there's no opportunity of being hired at stage 3. It implies  $w(2) = \alpha v (> 0)$ . If the worker has strictly positive probability of being hired at stage 2, then  $R(1) > 0$ . If not,  $R(1) = 0$ . In total,  $R(1) \geq 0$ , which implies  $w(1) \geq \alpha v = w(2)$ . ■

##### A2. Proof of Lemma 2

$L/J > m$  implies each firm interviews the density  $m$  of workers at stage 1. Therefore, at stage 2, each firm matches with the density  $L/J - mpq$  of workers. Since  $m \geq L/J - mpq$ , no firm conducts screening at stage 2. Then, the screening policy at stage 1 does not change the firm's monetary payoff. The prejudiced firms select  $a$  and the unprejudiced firms select  $r$ . ■

### A3. Proof of Proposition 1

Since  $m \geq L/J - mpq$ , each unemployed worker at stage 2 have an opportunity of interview. Then  $R(1) = q\alpha v$ , which leads to  $w(1) = \alpha v\{1+q(1-\alpha)\} > w(2) = \alpha v$ . Since size  $D$  of prejudiced firms conduct the screening policy  $a$ , for each of them,  $l_a(1)/l_b(1) > A(1)/B(1) = A/B$ . For each unprejudiced firm,  $l_a(1)/l_b(1) = A(1)/B(1)$ . Naturally, the type  $a$  agents earn  $w(1)$  with higher probability than the type  $b$  agents. Since more type  $a$  agents are hired at stage 1 than type  $b$  agents,  $l_b(2)/l_a(2) = B(2)/A(2) > B(1)/A(1) = B/A$ . ■

### A4. Proof of Lemma 3

Define  $A_k$  and  $B_k$  as the total size of type  $a$  and  $b$  workers who are hired at stage  $k$ . Then, routinely, the following values can be calculated:

$$\begin{aligned} A_1 &= \min(m, A/J) \cdot pqD + (Am)/(A+B) \cdot pq(J-D), \\ B_1 &= \max(0, m - A/J) \cdot pqD + (Bm)/(A+B) \cdot pq(J-D), \\ A_2 &= (A - A_1) \cdot (Ap - A_1)/(A - A_1) \cdot q = (Ap - A_1)q, \\ B_2 &= (B - B_1) \cdot (Bp - B_1)/(B - B_1) \cdot q = (Bp - B_1)q, \\ L_a^* &= A_1 + A_2 = Apq + A_1(1 - q), \quad \text{and} \quad L_b^* = B_1 + B_2 = Bpq + B_1(1 - q). \end{aligned}$$

It can be easily shown that  $A_1/A > B_1/B$ , which implies  $L_a^*/A > L_b^*/B$ . ■

### A5. Proof of Proposition 2

The first condition implies the screening sequentially occurs at both stages 1 and 2. The second condition implies that the density of type  $b$  workers whom one firm matches with is less than the manpower limit,  $m$ . Under these conditions, type  $a$  workers have positive probability of being interviewed at stage 2, which establishes  $w(1) > w(2)$  for type  $a$  workers. This inequality, in turn, is necessary for the prejudiced firms to conduct the discriminatory screening.

As is shown in A4, the prejudiced firms' discriminatory screening decreases the proportion of potentially qualified workers in type  $a$  workers as a whole more severely than type  $b$  workers ( $(Bp - B_1)/(B - B_1) > (Ap - A_1)/(A - A_1)$ ). Therefore, at stage 2, all the firms conduct the screening  $b$  in order to interview as many workers with higher probability of qualification as possible. This screening, in turn, makes  $R(1)$  of type  $b$  worker higher than that of type  $a$  worker. Then, if a type  $b$  worker gets hired at stage 1, his/her wage is higher than that for type  $a$  worker hired at stage 1. This drives the unprejudiced firms to take the policy  $a$  at stage 1. ■

### A6. Proof of Lemma 4

Similarly as in A4, the author derives the (implicit) value of  $L_a^*$  and  $L_b^*$ :

$$L_a^* = A_1 + A_2 = Apqr + A_1(1 - q\gamma), \quad \text{and} \quad L_b^* = B_1 + B_2 = Bpq + B_1(1 - q),$$

$$\begin{aligned} \text{where} \quad A_1 &= \min(m, A/J) \cdot pqD + (Am)/(A+B) \cdot pq(J-D), \\ B_1 &= \max(0, m - A/J) \cdot pqD + (Bm)/(A+B) \cdot pq(J-D), \\ \gamma &= \{mJ - (B - B_1)\}/(A - A_1) < 1. \end{aligned}$$

It can be shown that  $B - B_1$  is increasing function of  $B \in (0, \infty)$  whose range is  $(0, \infty)$ . Then, decreasing  $B$  moves  $\gamma$  close to 1. If  $\gamma = 1$ , A4 of the Appendix ensures  $L_a^*/A > L_b^*/B$ . If simultaneously  $L$  is sufficiently close to  $m(1 + pq)J$ , since  $mJ - (B - B_1)$  is close to  $A - A_1$ ,  $\gamma$  can be sufficiently close to 1, so that  $L_a^*/A > L_b^*/B$  still stands due to continuity.

On the other hand, if  $\gamma = 0$  and  $q = 1$ ,  $L_b^*/B = p > A_1/A = L_a^*/A$ . Then, if  $B - B_1$  is sufficiently close to  $mJ$  and  $q$  is close to 1,  $L_b^*/B > L_a^*/A$  still stands, again by continuity. ■

### A7. Proof of Lemma 5

The equilibrium  $(a, b)$ 's robustness can be routinely proved. For the equilibrium  $(b, a)$ , the robustness is equivalent to a pair of conditions:  $L_b^e(1)/L_a^e(1) > B/A$  and  $L_a^e(2)/L_b^e(2) > A^e(2)/B^e(2)$ .

Here, the prefix  $e$  in  $L_k^e(t)$ ,  $A^e(2)$ , and  $B^e(2)$  indicates that each value is evaluated with the prejudice backrush granted. Then, the two conditions are solved as below:

$$\begin{aligned}\epsilon_1 &\equiv \frac{J \min\{m, B/J - B/A \cdot (m - B/J)\}}{D \min\{mB/A, (A+B)/J - m\} + J \min\{m, B/J - B/A \cdot (m - B/J)\}} > e, \\ \epsilon_2 &\equiv \frac{JA^e(2)}{DB^e(2) + JA^e(2)} > e, \quad \text{where} \\ A^e(2) &= A - Jpq \max\left\{0, m - \frac{B}{J}\right\} + \left[ J \min\left\{\frac{mA}{A+B}, \frac{B}{J} - \frac{mB}{A+B}\right\} + D \min\left\{\frac{mB}{A+B}, \frac{A}{J} - \frac{mA}{A+B}\right\} \right] pqe, \\ B^e(2) &= B - Jpq \min\left\{m, \frac{B}{J}\right\} + \left[ J \min\left\{\frac{mA}{A+B}, \frac{B}{J} - \frac{mB}{A+B}\right\} + D \min\left\{\frac{mB}{A+B}, \frac{A}{J} - \frac{mA}{A+B}\right\} \right] pqe.\end{aligned}$$

The left hand side of first inequality is explicitly a decreasing function of  $D$ . Although the second one is still an implicit inequality, its left hand side has a rather simple form since the author has assumed  $(m > A(2)/J) \wedge (m > B(2)/J)$ : the relevant coexistence of two equilibria. The author transforms it into a quadratic inequality as below:

$$\begin{aligned}(J - D)[J \min\{mA/(A+B), -mB/(A+B) + B/J\} + D \min\{mB/(A+B), A/J - mA/(A+B)\}]pqe^2 \\ - [J[J \min\{mA/(A+B), -mB/(A+B) + B/J\} + D \min\{mB/(A+B), A/J - mA/(A+B)\}]pq \\ + JA + DB - J^2mpq + (J - D)Jpq \min\{m, B/J\}]e \\ + JA - J^2pq[m - \min\{m, B/J\}] > 0.\end{aligned}$$

Let the lower boundary of solution range be  $\epsilon_2$ . Since  $e \in (0, 1)$ , the left hand side is decreasing to  $D$ , which implies  $\epsilon_2$  is also decreasing. Defining  $\epsilon^* \equiv \min\{\epsilon_1, \epsilon_2\}$ ,  $\epsilon^*$  is decreasing to  $D$ .

### A8. Proof of Proposition 3

The survival of the prejudiced firm is obvious because it maximizes its profit similarly as its unprejudiced counterparts. It does not depend on the level of  $D/J$ . The outcomes ND, DOE, and DWEa or DWEb appear when  $J \geq L/m$ ,  $L/m > J \geq L/\{m(1 + pq)\}$ , and  $L/\{m(1 + pq)\} > J$ , respectively. A firm's (gross) monetary payoff in the case of ND with  $J = L/m$  is  $\pi_1 \equiv mpq(1 - \alpha)(2 - q - \alpha q)v$ . A firm's (gross) monetary payoff in the case of DOE with  $J = L/\{m(1 + pq)\}$  is  $\pi_2 \equiv mpq(1 - \alpha)(2 + pq - q - \alpha q)v$ . These two values are the monetary payoff thresholds for the equilibria of ND and DOE, respectively. If  $\kappa \in (0, \pi_1]$  (*resp.*  $\kappa \in (\pi_1, \pi_2]$ ), in the market with free entry, the outcome of ND (*resp.* DOE) realizes. (Proving the nonincreasingness to  $J$  of firm-wise gross monetary payoff is dedicated to the readers.) ■

## References

1. Arrow, K.J. (1973), "The Theory of Discrimination", in 'Discrimination in Labor Markets' (ed. Ashenfelter, O. and Rees, A.), Princeton University Press, 3-33.
2. Becker, G.S. (1953), "The Economics of Discrimination", University of Chicago Press.
3. Black, D.A. (1995), "Discrimination in an Equilibrium Search Model", *Journal of Labor Economics*, vol. 13, 309-334.
4. Coate, S. and Loury, G. (1993), "Will Affirmative-Action Policies Eliminate Negative Stereotypes?", *American Economic Review*, vol. 83, 1220-40.
5. Epstein, R.A. (1992), "Forbidden Grounds: The Case Against Employment Discrimination Laws", Harvard University Press.
6. Fryer, R.G.Jr. (2007), "Implicit Quotas", forthcoming in *Journal of Legal Studies*.