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Abstract

The present paper proposes a theory of man, wherein man constructs models of the world based on past experiences in social situations. The present theory considers experiences, or chunks of impressions, as primitives instead of an "objective" game, which is assumed to be given in the standard game theory. Agents construct models of the world based on direct and indirect experiences. Each model comprises a structural part and a factual part. The structural part is represented as a game, while the factual part is represented as a strategy profile of this game. In constructing a model, an agent might use certain axioms, for example, coherence, according to which the model should be able to explain his or her own experiences; conformity to a solution concept; and minimality with respect to some simplicity measure. A few applications are presented to demonstrate how this theory works.

JEL: B4, C79, Z19

Keywords: Induction, Model, Experience, Axiom, Model Construction

We are what we think.

All that we are arises with our thoughts.

With our thoughts we make the world.

"The Dhammapada: the saying of the Buddha"

1 Introduction

This paper proposes a theory of man, wherein man constructs models of the world based on past experiences in social situations. The present theory considers experiences, or chunks of impressions, as primitives instead of an "objective" game, which is assumed to be given in the standard economic theory. Agents construct models of the world based on direct and indirect experiences. Each model comprises a structural part and

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a factual part. The structural part is represented as a game, while the factual part is represented as a strategy profile of this game. In constructing a model, an agent might use certain axioms, for example, coherence, according to which the model should be able to explain his or her own experiences; conformity to a solution concept; and minimality with respect to some simplicity measure.[1]

For more than a century, differences in intellectual ability between human being and other species have been extensively studied (see, e.g., Thorndike (1911/2000) for some earlier works). Many "intelligent" activities, especially those analyzed by Simon (1957), are now known to be shared not only by primates but also by a variety of animals. Many birds and mammals are known to use their intelligence to try to behave satisfactorily, if not optimally, in various situations. They too learn how to hunt, fly, and breed. For example, it is commonly observed that birds bred by humans can neither fly nor breed by themselves.[2] Moreover, there was a bird, an African grey parrot, that had been claimed to demonstrate numerical competence.[3] Needless to say, humans use their instinct, like other animals, to avoid danger and react to certain stimuli. Tendencies of such behavior have been extensively studied in psychology and, more recently, behavioral economics in the context of strategic interaction (see, e.g., Camerer (2003)).

Nevertheless, humans are distinct from other species with respect to the manner in which they use intelligence. One of the intellectual activities that are often observed in humans, but not in other animals, is the construction of a model of the world that explains their experiences.[4] Focusing on the observation that experiences play a major role in shaping the human mind, this paper proposes a formal game-theoretical framework to study such activities of humans in social situations.

The difference between the standard theory and the present theory is summarized in Figure 1. The standard game theory takes a model, or the structure of a game, as

[1] Peirce (1898/1992) called this activity *retroduction* (or abduction), claiming that we had to distinguish this activity from the "standard" induction by which we enlarge our observation from samples to the entire population. I am grateful to Takashi Shimizu for pointing this out to me. See Matsui and Shimizu (2007) for more discussion.

[2] There are numerous reports on the difficulty of animals' returning to the wild. Many programs are designed to teach animals various skills to survive in the wild. See, e.g., Hendron (2000).

[3] For example, Pepperberg (1994) reported that an African gray parrot (Psittacus erithacus), Alex, trained to label vocally collections of 1-6 simultaneously presented homogeneous objects, correctly identified, without further training, quantities of targeted subsets in heterogeneous collections. For each test trial Alex was shown different collections of 4 groups of items that varied in 2 colors and 2 object categories (e.g., blue and red keys and trucks) and was asked to label the number of items uniquely defined by the conjunction of 1 color and 1 object category (e.g., "How many blue keys?"). The collections were designed to provide maximal confounds (or distractions). Unfortunately, further tests cannot be conducted since Alex died on September 6, 2007, at the age of 31.

[4] However, it is difficult to reject the hypothesis that animals, too, perform such an intelligent activity of constructing a model of the world, in a broad sense.

given and applies a solution concept such as Nash equilibrium or a behavior rule to the model in order to derive the strategies/behavior of the players of the game. In the sense that a specific act is induced by a general principle, the present paper calls this theory the *deductive game theory*.

On the other hand, the present theory takes experiences, or chunks of impressions, as primitives. Based on them, an agent constructs a model. Some axioms are used in constructing a model. In the sense that a general structure of the game is induced based on limited experiences, the present paper calls this theory *inductive game theory*.

Model $\Rightarrow$ Behavior	Experiences $\Rightarrow$ Model
$\uparrow$	$\uparrow$
Solution concept	Axioms
Deductive (game) theory	Inductive (game) theory

Figure 1: Deductive and Inductive Theories

Three remarks are in order. First, these two theories should not be regarded as substitutes, rather, they are complements. In reality, people use both induction and deduction in accumulating knowledge, which eventually affects their behavior. Second, the construction of a model does not have to precede action taking. Rather, experiences typically include those impressions that are obtained through one's own behavior. Third, the present framework takes game theory as a language agents use to describe the world rather than a refutable "theory". Although the present way of describing models in the language of game theory is far from being general, it enables us a rigorous study of the situations agents are in.

Several applications are presented to show the basic workings of the theory. The first application is concerning entry and predation. The failure of Air Do, an airline company, illustrates the workings of the theory. The second one is on bullying. Through the activity of bullying in school, children may construct a specific way of viewing the situation. The third application is a simple repeated interaction between two agents, say, a wife and a husband. Two might view the situation quite differently after, say, a defection of the wife, and the difference may be the source of the impossibility of renegotiation. The fourth application is concerning discrimination and prejudices as discussed in Kaneko and Matsui (1999). Instead of the standard argument on this subject (see Becker (1957)), i.e., that prejudices lead to discrimination, the present framework allows an argument that the fact of segregation gives rise to prejudices against the segregated. The fifth application is concerning the importance of pioneers.

The idea of the construction of models by agents based on experiences was initiated by Kaneko and Matsui (1999), who examined a specific game called the festival game. Subsequent papers by Kaneko and Kline (2006) and Matsui and Shimizu (2007) are closely related to the present paper. Kaneko and Kline (2006) proposed the concept of information protocol and demonstrated a correspondence between games expressed in extensive form and in information protocols.

Matsui and Shimizu (2007) confined their attention to the class of repeated games and sought conditions under which an objective game and a subjectively constructed model coincide. The present paper does not presume the existence of an objective game, and therefore, does not pay attention to the conditions under which agents can reconstruct the objective game from experiences.

This paper focuses on induction as the main inference rule. In this regard, the present paper shares a common thread with a sequence of works by Gilboa and Schmeidler (1995, 2001), Fudenberg and Levine (1993), and Battigalli and Siniscalchi (2002). However, a critical difference is that their focus is on the decision making process, whereas the present paper focuses on man's creation of models of the world. Referring to Peirce (1898/1992), Matsui and Shimizu (2007) argued that the kind of activity analyzed in the present paper should preferably be called *retroduction*, or the *inference to the best explanation*, as subsequent researchers have called it.

The rest of the paper is organized as follows. Section 2 presents the basic framework. Section 3 studies several applications. Section 4 concludes the paper.

2 The Framework

Let us first denote by $\mathcal{N}$ the set of all possible agents, by $\mathcal{A}$ the set of all possible acts, by $\mathcal{O}$ the set of all possible emotions, and by $\mathcal{I}$ the set of all possible impressions, of which meaning will be clear below. We assume that $\mathcal{N}$, $\mathcal{A}$, $\mathcal{O}$, and $\mathcal{I}$ are mutually disjoint.

2.1 Impressions and Experiences

Agents accumulate experiences, each of which constitutes a chunk of impressions sensed and felt by the agent. Let $\mathcal{I}$ be the set of impressions, which are primitives of the current framework.

Formally, an *experience* ε_i of Agent i ($i \in \mathcal{N}$) is a finite set of *impressions* $\omega_{i1}, \ldots, \omega_{iL} \in \mathcal{I}$, i.e., $\varepsilon_i = \{\omega_{i1}, \ldots, \omega_{iL}\}$. We denote by $\mathcal{E}$ the set of all possible experiences, i.e., all finite sets of impressions.

This setup is sufficiently general since the set $\mathcal{I}$ of impressions is arbitrary. Yet, among various forms of impressions, the following forms along with their intended meanings are of special attention:

(i) $N \subset \mathcal{N}$: agents in N meet each other;

(ii) $(j : A_j) \in \mathcal{N} \times 2^{\mathcal{A}}$: Agent j has a set A_j of available acts;[5]

(iii) $(j : a) \in \mathcal{N} \times \mathcal{A}$: Agent j takes an act a;

(iv) $(j : \text{“}\textit{emotion}\text{”}) \in \mathcal{N} \times \mathcal{O}$: Agent j expresses or feels an “*emotion*”;

(v) $\varepsilon \succeq_i \varepsilon'$: Agent i weakly prefers experience ε to ε' (we also use $\varepsilon \succ_i \varepsilon'$ and $\varepsilon \sim_i \varepsilon'$ to mean strict preference and indifference, respectively).

(vi) $\emptyset$: a *null experience*.

We assume that these forms are in $\mathcal{I}$. We identify a sequence of experiences $(\varepsilon^1, \ldots, \varepsilon^{s-1}, \emptyset, \varepsilon^{s+1}, \ldots, \varepsilon^S)$ with $(\varepsilon^1, \ldots, \varepsilon^{s-1}, \varepsilon^{s+1}, \ldots, \varepsilon^S)$.[6]

Some examples of experiences are given below:

- $\varepsilon_i = (\{\{i, j, k\}, (i : A_i)\}, \{(i : a), (j : b)\}\}$: Agent i observed that Agents i, j, k met, that i has acts in A_i available, that i took a, and that j took b, but did not observe the act of k;
- $(\varepsilon_i^1, \varepsilon_i^2, \varepsilon_i^3)$ with $\varepsilon_i^1 = \{(i : a), (i : \text{“}\textit{pain}''))\}$, $\varepsilon_i^2 = \{(i : b), (i : \text{“}\textit{fun}''))\}$, $\varepsilon_i^3 = \{(i : a), (i : \text{“}\textit{calm}''), \varepsilon_i^1 \succ_i \varepsilon_i^2\}$: Agent i felt ”pain” when i took a, ”fun” when b, ”calm” when c, and i thought, when he was ”calm”, ε_i^1 was preferred to ε_i^2 in retrospect;

[5] This type of element is less obvious than others. In reality, what I can observe is the fact that, say, someone opened the door of my office and walked toward me, and I do not observe that the person had an option of staying home and watched TV programs. I do not observe the latter, but based on my past experiences, I am convinced that he could have stayed home instead of coming to my office. Nonetheless, the subsequent setup assumes that this class of observation is also in $\mathcal{I}$ for the matter of convenience.

[6] If one would like to incorporate the notion of time, one can do it by, say, adding time to an experience.

- $\varepsilon_i = \{\{j, k\}, (j : a), (k : ☹)\}$: Agent i observed that Agents j and k met, that j took a, and k expresses ☹.

2.2 Models

We use stochastic games as models of the world agents construct. Based on the set of experiences, each agent constructs a *model*, which represents his understanding of the situation in question. A *model* is generically given by

$$m = \langle (N, G, \mu), \sigma \rangle = \langle (N, G, \mu), (\sigma_i)_{i \in N} \rangle,$$

where (N, G, μ) is the *structural* part of the model, which is represented as a (modified) stochastic game, and σ is the *factual* part of the model, which is represented as a strategy profile of the stochastic game. Here, we have the following:

- $g = \langle N^g, (A_i^g)_{i \in N^g}, (u_i^g)_{i \in N}, (\varphi_i)_{i \in N} \rangle$ $(g \in G)$ is an augmented game in strategic form where
  - $N^g \subset N$ is the set of agents,
  - A_i^g is the set of acts of Agent $i \in N^g$;
  - $u_k^g : A^g \equiv \times_{j \in N^g} A_j^g \to \mathbb{R}$ is the payoff function of Agent $k \in N$;
  - φ_i^g $(i \in N)$ is an *experience function* of Agent i that maps $\{\emptyset\} \cup A^g$ into $\mathcal{E}$;
- μ is a *transition function* that maps each (g, a) to a probability distribution over G; and
- σ_i $(i \in N)$ is a *strategy* or a *behavior rule* of Agent i, which is a function of the past $\varphi_i(\cdot)$'s.

In this description, $\varphi_i^g(\emptyset)$ is an experience of i before g is played. The value $\varphi_i^g(\emptyset)$ typically, *albeit* not necessarily, contains the set of agents who meet to play the game and the set of available acts. Let $\mathcal{M}$ be the set of all such models.

Some models are of special interest. Here, we mention two classes of them. The first class is that of repeated game models. From sunrise to everyday work, one often views the situation he or she faces as if it would repeat indefinitely.

Model 1 A model $m = \langle (N, G, \mu), \sigma \rangle$ is an *infinitely repeated game model with discounting* if G is a singleton with $N^g = N$ (*a fortiori*, and μ is an identity map. In particular, it is a repeated game model with *perfect monitoring* if, for all $i \in N$, $\varphi_i^g(\emptyset) = \{N\}$ and $\varphi_i^g(a) = \{(j : a_j)\}_{j \in N}$.

Model 2 A model $m = \langle (N, G, \mu), \sigma \rangle$ is a *pairwise and uniform random matching model* (henceforth, *random matching model*) if G consists of g_{ij}'s ($i, j \in N$) where $g_{ij} = \langle \{i, j\}, (A_i, A_j), (u_k)_{k \in N}, (\varphi_k)_{k \in N} \rangle$, $\mu(\cdot, \cdot)(g_{ij}) = 1/|N|(|N| - 1)$. In particular, it is a random matching model with *full observation* if each agent observes agents' identity and act pair taken for every game, i.e., $\varphi_i(g_{jk}) = \{\{j, k\}\}$ and $\varphi_i(g_{jk}, (a_j, a_k)) = \{(j : a_j), (k : a_k)\}$. On the other hand, it is a random matching model with *private observation* if each agent observes agents' identity and act pairs taken only for the games in which this agent participates, i.e.,

$$\varphi_i(g_{jk}) = \begin{cases} \{\{j, k\}\} & \text{if } i \in \{j, k\}, \\ \emptyset & \text{otherwise.} \end{cases}$$

and

$$\varphi_i(g_{jk}, (a_j, a_k)) = \begin{cases} \{(j : a_j), (k : a_k)\} & \text{if } i \in \{j, k\}, \\ \emptyset & \text{otherwise.} \end{cases}$$

2.3 Axioms

Axioms are the criteria which agents use to construct models of the world. There is no axiom that *ought* to be used *a priori*. Axioms themselves may be in flux in human mind, just like a researcher adopting different axioms from time to time. However, there are some that are considered plausible. The first of such axioms is *coherence*, which requires that a model be able to explain one's experiences.

Axiom 1 (Coherence) Given a model $m = \langle (N, G, \mu), \sigma \rangle$ and a sequence $\tilde{\varepsilon}_i = (\tilde{\varepsilon}_i^1, \dots, \tilde{\varepsilon}_i^K)$ of experiences, m is said to be coherent with $\tilde{\varepsilon}$ if there exist $\varepsilon_i = (\varepsilon_{ia}^t, \varepsilon_{ip}^t)_{t=0}^T$ that is equivalent to $\tilde{\varepsilon}_i$, $(g^0, g^1, \dots, g^T)$, and $(a^0, a^1, \dots, a^T)$ such that the following conditions hold:

1. $\mu(g^{t-1}, a^{t-1})(g^t) > 0$, $t = 1, \dots, T$;
2. $\sigma_j(\varphi_j^{g^0}(\emptyset), \varphi_j^{g^0}(a^0), \dots, \varphi_j^{g^{t-1}}(a^{t-1}), \varphi_j^{g^t}(\emptyset))(a_j^t) > 0$, $t = 1, \dots, T$, $j \in N$;
3. $\varphi_i^{g^t}(\emptyset) = \varepsilon_{ia}^t$, and $\varphi_i^{g^t}(a^t) = \varepsilon_{ip}^t$, $t = 0, \dots, T$.
4. $u_i^{g^t}(a^t) \geq u_i^{g^{t'}}(a^{t'})$ if there exists $\tau = 0, 1, \dots, T$ such that $(\varepsilon^t \succeq_i \varepsilon^{t'}) \in \varepsilon^\tau$ holds where "$>$" and "$=$" hold for "$\succ_i$" and "$\sim_i$", respectively.

Also, we may add another condition to consider the notion of statiscal coherence. Given a set of statistical tests and a sequence of experiences, a model is *statistically* *coherent* if, in addition to the four conditions of Axiom 1, the null hypothesis that the system is governed by μ is not rejected by these tests. We do not define this axiom more rigorously as the way it is defined depends upon the set of statistical tests to be used. We do not use statistical coherence in the subsequent applications.

A *solution concept* is a correspondence ψ that maps a stochastic game to a set of strategy profiles (possibly empty for some games). It is defined without referring to experiences.

Axiom 2 (Conformity) Given a solution concept ψ, a model $m = \langle (N, G, \mu), \sigma \rangle$ conforms to the behavior rule ψ if $\sigma \in \psi(N, G, \mu)$.

An example of solution concepts is Nash equilibrium. Another is solution by backward induction. Other concepts induced by, say, some behavior rules can be represented as a ψ, too.

Axiom 3 (Uniqueness of Outcome/Solution) Given a model $m = \langle (N, G, \mu), \sigma \rangle$, a solution concept ψ induces the *unique outcome* if all $\tilde{\sigma}$'s in $\psi(N, G, \mu)$ induce the same stochastic process of outcome. ψ induces the *unique solution* if $\psi(N, G, \mu) = \{\sigma\}$.

The following two axioms are controversial in science. Nonetheless, there are tendencies to use them in reality by scientists as well as by laymen. The first one is the principle of simplicity and the second is that of observability.

Given $M \subset \mathcal{M}$, let $\geq_M$ denote a binary relation on M. We write $m >_M m'$ if $m \geq_M m'$ holds, but not $m' \geq_M m$.

Axiom 4 (Minimality) Given $M \subset \mathcal{M}$ and a binary relation $\geq_M$ over M, a model m is said to be *minimal* with respect to $\geq_M$ on M if there exists no $m' \in M$ satisfying $m >_M m'$.

Different agents may use different binary relations. A confused agent may have an intransitive binary relation, but we may assume that $\geq_M$ is a partial preorder.[7]

The next axiom is a principle of observability, according to which one tends to choose a model that explains situations, especially utility functions, only by observables. Given a model $m = \langle (N, G, \mu), \sigma \rangle$, a *variable* is a function X that maps each (g, a) to an element of $\mathcal{O}$. Also, an *observable variable* for player $i \in N$ is a variable X such that $X(g, a) \subset \varphi_i(g, a)$ for all $g \in G$ and all $a \in \{\emptyset\} \cup A^g$, and an observable variable is a observable variable for some player i.

[7] A binary relation $\geq_M$ is a *partial preorder* if it satisfies reflexivity and transitivity, i.e., $[\forall x \in M(x \geq_M x)]$ and $[\forall x, y, z \in M(x \geq_M y \& y \geq_M z \Rightarrow x \geq_M z)]$, respectively.

Axiom 5 (Observability) Given a sequence $(\varepsilon_i^1, \ldots, \varepsilon_i^K)$ of experiences, a model $m = \langle (N, G, \mu), \sigma \rangle$ satisfies the *principle of observability* if u^g $(g \in G)$ and σ are the functions of observable variables.

2.4 Prior Beliefs

Prior to the construction of a model based on experiences, agents may have held a certain belief of the situation. At this point, we do not care where this belief comes from, e.g., whether it comes from pure reasoning or from prior experiences. This belief may take various forms. A possible representation of such a belief is to restrict a possible class of models, at least from the viewpoint of researchers. Let $M \subset \mathcal{M}$ be the subset of games in extensive form. An agent's a priori knowledge can be represented by such an M.

3 Applications

3.1 Predation

Experience is a dear teacher, but fools will learn at no other.

–Benjamin Franklin

In 1998, after raising money from the general public, Air Do entered the Japanese domestic airline market after the deregulation of the airline industry in 1990s. Air Do was one of Japan's first low-fare airlines, operating between Chitose, Hokkaido and Haneda, Tokyo. Initially called "*Do-min no Tsubasa* "(The wing of Hokkaido-residents), it provided its passengers with low-fare flights between Tokyo and Hokkaido. It competed with Japan's major domestic carriers (All Nippon Airways (ANA), Japan Airlines, and Japan Air System), which lowered their fares to Air Do's level without extensively compromising on corporate profits. After two years of incurring losses and despite continuous financial support from the local government of Hokkaido, Air Do went bankrupt, retired all its stocks, and entered into a code-sharing agreement with ANA. Not only did Air Do lost its money, but also lost its dream of becoming "the wing of Hokkaido-residents," adopting the same general fare structure as the major airlines. ANA seems to have emerged as the winner of this predation game as it acquired Air Do as a low cost airline. Indeed, ANA has had Air Do expand its routes from only one (Haneda-Chitose) to four (Haneda-Chitose, -Asahikawa, -Hakodate, and -Memanbetsu).

To understand this situation, suppose that a potential entrant E considers whether or not to enter a market monopolized by an incumbent I before deregulation. From some other markets of similar characteristics, E learns that once an entrant enters, an incumbent often acquiesces, and the two firms share the market accordingly. Formally, assume that an Entrant E's (indirect) experiences concerning, for instance, the US airline market, are

$$(\mathit{Regulation}, (E' : \{not\}), (E' : not), (I' : \pi^m_{I'}), (E' : 0)),$$

before deregulation, and

$$(\mathit{Deregulation}, (E' : \{not, enter\}), (E' : enter), (I' : \{p_H, p_L\}), (I' : p_H), \\ (I' : \pi^d_{I'}), (E' : \pi^d_{E'}), (\pi^m_{I'} > \pi^d_{I'}), (\pi^d_{E'} > 0)),$$

after deregulation, where $(E' : \{not, enter\})$ implies that E' has the two options, "enter" and "not enter"; $(E' : enter)$ implies that E' chose to "enter"; $(I' : \{p_H, p_L\})$ implies that I' had options of p_H and p_L; $(I' : p_H)$ implies that I' took p_H; $(I' : \text{“2”})$ implies that I' obtained the payoff of 2; and $(E' : \text{“1”})$ implies that E' obtained 1. In this description, p_H (resp. p_L) denotes a high (resp. low) price.

Having observed them, E may construct a structural model (N, G, μ) after deregulation as follows:

- $N = \{I, E\}$;
- $G = \{g^m, g^d\}$ where m and d stand for monopoly and duopoly, respectively;
  - $N^m = \{E\}$, $N^d = \{I\}$;
  - $A^m_E = \{enter, not\}$;
  - $A^d_I = \{p_H, p_L\}$;
  - $u^m_I(\cdot) = 4,\ u^m_E(\cdot) = 0$;
  - $u^d_I(p_H) = 2,\ u^d_E(p_H) = 1$;
  - $u^d_I(p_L) = v,\ u^d_E(p_L) = w$ for some $v, w \in \mathbb{R}$;
- $\mu(g^m, a) = \begin{cases} 1_{g^m} & \text{if } a_E = not, \\ 1_{g^d} & \text{if } a_E = enter, \end{cases}$

  where 1_g is a probability distribution that assigns one to g;
- $\mu(g^d, a) = 1_{g^d},\ \forall a \in A^d_I$.

At the same time, the factual part of the model is given as follows:

- $\sigma_I(\cdot, g^d) = p_H$,
- $\sigma_E(\cdot, g^m) = enter$.

To intuitively understand the above model, it may be helpful to consider an "isomorphic" game in extensive form, though the term "isomorphic" is not formally defined. In order to construct a model "isomorphic" with the one created by E, let us assume, for the moment, that I has two options, p_L-*forever* and p_H-*forever* labelled *fight* and *acquiesce*, respectively.

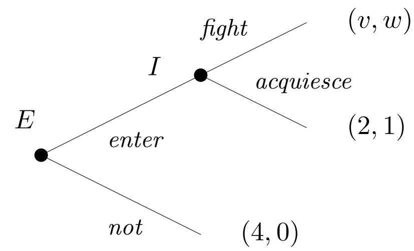

Figure 2: Predation Game

This game is coherent with E's experiences no matter what v and w may be. Also, suppose that $\geq_{|\cdot|}$ is a partial order with respect to the sizes of N, G, and A_i^g's.[8] Then this model is a minimal model with respect to $\geq_{|\cdot|}$.

Suppose further that ψ complies with backward induction, then E's experience (*enter*, *acquiesce*) implies that $2 > v$ holds, which leads to a positive profit for E even if w is negative.

On the other hand, I constructed a different structural model which is the same as E's structural model except that g^d moves to g^p if "fight" is chosen by I, and in g^p, E decides whether to "exit" or "stay". In this new model, $A_E^{g^p} = \{stay, exit\}$ and

$$\mu(g^p, a) = \begin{cases} 1_{g^p} & \text{if } a_E = stay, \\ 1_{g^m} & \text{if } a_E = exit, \end{cases}$$

in place of $\mu(g^d, a) = 1_{g^d}$ with $u_I^{g^p}(\cdot) = v$ and $u_E^{g^p}(\cdot) = w$.

An "isomorphic" game with the model created by I can be constructed if we assume that after I takes *fight*, E has an option of *exit*, and otherwise *stay*.

[8] Precisely speaking, one may write $(N, G, \mu; \sigma) \geq (N', G', \mu', \sigma')$ if $|N| \geq |N'|$, $|G| \geq |G'|$, and there exist one-to-one (but not necessarily onto) correspondences $\varphi : N' \to N$ and $\rho : G' \to G$ such that for all $g \in G'$ and all $i \in N'^g [|A_{\varphi(i)}^{\rho(g)}| \geq |A_i^g|]$.

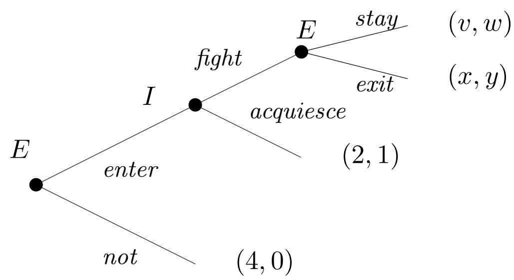

Figure 3: Predation Game with an Exit Option

In this game, E has to be cautious and in fact refrain from entry since it should expect a negative profit if it believes that ψ complies with backward induction.

In the case of Air Do, it could not bear the loss caused by the predatory pricing of the two incumbents, ANA and JAL, and exited the market, or to be precise, retired its capital and reached a code-share agreement with ANA.

3.2 Bullying

> "You will probably be bullied wherever you may go unless you have some fighting spirit." –Shintaro Ishihara[9]

Suppose that there are four children, A, B, C, and D, which has already been part of prior knowledge. Also, every child has prior knowledge that each time two children meet in pairs, they simultaneously decide whether to play friendlily (F) or unfriendlily (U). Suppose now that these children have observed that each time two of A, B, and C met in pairs, they took F and looked happy, while when they met D and formed a pair, they played U, while D played both F and U from time to time, and the two looked unhappy. In addition to these impressions, the children observed various attributes of each other, e.g., color, height, body shape, face, etc. Let ε_j denote such an experience of Child j ($j = A, B, C, D$).

There are numerous models that are coherent with the above experience even if we restrict our attention to random matching models. Here, we consider two classes. The first one assumes some intrinsic differences between the children, whereas the second does not. In the both models, Nature randomly determines a pair of children to be matched. After two children are matched, they play a simultaneous move game where both of them have two available acts F (Friendly) and U (Unfriendly), which the children are also aware of.

[9] A remark at press conference on Nov. 10, 2006; translated by the author

In the first model of Child $i = A, B, C$, a utility function can be of the following form:

$$u_i(a_i, a_j; j) = \begin{cases} 1 & \text{if } a_i = a_j = F \text{ and } j \neq D, \\ 0 & \text{otherwise, } (i = A, B, C). \end{cases} \tag{1}$$

Behavior rules of A, B, and C are given by:

$$\sigma_i(\{i, j\}) = \begin{cases} F & \text{if } j \neq D, \\ U & \text{if } j = D, \ (i = A, B, C). \end{cases}$$

This model of Child i is coherent with ε_i, and their strategy profile constitutes a Nash equilibrium. Moreover, under some "reasonable" criteria of minimality such as the one that counts "complexity" by the number of acts and payoff values, this model becomes minimal.

If it happens to be the case that D is taller than the other three children, then children may construct a model in such a way that they do not enjoy playing with a tall child. To construct such a model, suppose that h_j ($j = A, B, C, D$) is the height of Child j, and that $h_j < \bar{h}$ for $j \neq D$, while $h_D > \bar{h}$ where $\bar{h}$ is a threshold value. In this case, we have

$$u_i(a_i, a_j; h_j) = \begin{cases} 1 & \text{if } a_i = a_j = F \text{ and } h_j < \bar{h}, \\ 0 & \text{otherwise, } (i = A, B, C). \end{cases} \tag{2}$$

in place of (1). The purpose of this analysis is to show that any attribute can be a reason for bullying.

In the second model, each child obtains one as a payoff if both choose F, and zero otherwise.

$$u_i(a_i, a_j) = \begin{cases} 1 & \text{if } a_i = a_j = F, \\ 0 & \text{otherwise, } (i = A, B, C, D). \end{cases} \tag{3}$$

Each child plays a repeated game strategy according to which they determine a "target" and play U whenever a child meets the target child, and they continue to do so until someone takes F against the target, after which the child who chose F now becomes a new target. This strategy profile is a subgame perfect equilibrium of the constructed repeated game.

3.3 Spilt Water

Fukusui Bon-ni Kaerazu.[10]

[10]Spilt water never returns to its tray, corresponding to the English saying, "There is no use crying over spilt milk."

–An old saying from the story of Tai Kung-Wang (*Taikobo*)

A well known chinese historical character, Tai Kung-Wang, or *Taikobo*, was left by his wife after reading and fishing day by day in spite of her devotion to her husband. When he became a local lord, the ex-wife tried to be reconciled with him. He said he would reinstate her if she could put spilt water back into the tray.

To study this story, we consider the following situation. In a repeated interaction, one defection sometimes devastates the relationship. To interpret such a situation, a game theorist would build a repeated game model of prisoners' dilemma given by Table 1 and assume that the players take the grim trigger strategies according to which one keeps cooperating (C) till the opponent defects (D), which triggers defection forever. Yet, some other people think that the game they play changes after they encounter defection.

		husband		
		C	D	R
wife	C	2, 2	-1, 3	0, -1
	D	3, -1	0, 0	0, -1
	R	-1, 0	-1, 0	-1, -1

Table 1: Prisoners' Dilemma

Suppose that there is a couple who get along well with each other. The wife thinks that the situation they are in can be represented by the repetition of the prisoners' dilemma given by Table 1. So far, their experiences were $((C, C), \ldots, (C, C))$. One day, the wife took D. He thought the husband would forgive him if he repented, which he actually did. However, it turns out that the husband claims "The game has been changed," and never takes C thereafter.

One may claim that what has been changed is the history, not the game. However, the following model m justifies the husband's claim that "the game has been changed". Denote by g^{PD} the game given by Table 1, and by g^B the game where the one who was betrayed has only one act D, which is given by Table 2.

		husband
		D
wife	C	-1, 3
	D	0, 0
	R	-1, 0

Table 2: Aftermath of Defection by Wife

Both models are coherent with their experiences. The two models are different, however, in terms of the possibility of the husband's changing his behavior to cooperate again after the defection of the wife. In the repeated game model, what the wife should do is to persuade the husband to take C again. Indeed, the husband's behavior is not "renegotiation-proof" in a loose sense. On the other hand, in the second model, the wife has to change the view of the world of the husband: his behavior together with D by the wife in Table 2 forms a renegotiation-proof Nash equilibrium.

3.4 Segregation

> I have a dream that one day on the red hills of Georgia, the sons of former slaves and the sons of former slave owners will be able to sit down together at the table of brotherhood.
>
> - Martin L. King, Jr.[11]

Suppose that in order to accommodate a person using a wheelchair, a store manager needs to set a ramp. If people using wheelchairs are segregated, not intentionally from the viewpoint of "normal" people, but by unconsciously creating barriers at different points, and consequently, if only a handful of these people make special efforts to come to town, the store manager may simply ignore their existence and may not build a ramp. If this is the case, it is inconvenient for people using wheelchairs, and therefore, they avoid coming to town unless they have a special interest or a wish to appeal an unfair treatment. In particular, if the people using wheelchairs feels that the store manager has been unfair to them, they must speak out against the discriminating behavior. This may give rise to tensions between the disabled and others. In fact, such store managers may construct a model in which the disabled create problems and think that they are uninvited guests. In turn, they might wish that the government to build more facilities so as to segregate them.

Kaneko and Matsui (1999) consider a festival game to study how the majority develops prejudices against the minority.[12] There are two sets of players N_1 and N_2 with $|N_1| > |N_2| + 1$, and two locations $L1$ and $L2$. This game has two stages. In the first stage, the players simultaneously choose a location to visit. In the second stage, after they observe the ethnicity configuration of their own location, i.e., whether or not people in N_i $(i = 1, 2)$ come to their location, they simultaneously choose either one of

[11] Quoted during a speech in Washington, D.C.

[12] Note that the current framework does not need such an underlying game.

the Friendly (F) and Unfriendly (U) actions. If one player chooses U, he/she obtains a default payoff of zero. If, on the other hand, one chooses F, his or her payoff depends upon the number m of players who take F in his or her location, which is given by

$$u(F, m) = m - \bar{m}.$$

We assume $\bar{m}$ is a non-integer number greater than two and less than $|N_2|$.

The festival game has various equilibria. The most efficient equilibrium is the one in which everyone goes to the same location. However, there is another type of equilibrium in which players in N_1 go to $L1$, those in N_2 go to $L2$, and each player takes F on the path. This equilibrium sustains itself if a player in N_2 goes to $L1$, not more than $|N_2| - 1$ people take F, and the rest of people in N_1 take U.

In this equilibrium, the experience of a player in N_1 is given by

$$\omega_F = (L1, \{1\}, F, |N_1| - \bar{m})$$

where the first term $L1$ indicates that he chose $L1$, $\{1\}$ is the observed ethnicity configuration, F is his own action, and $u(F, |N_1|) = |N_1| - \bar{m}$ is his or her payoff. On the other hand, one's experience when he or she observed N_2 becomes

$$\hat{\omega}_F = (L1, \{1, 2\}, F, x)$$

where x is at most $|N_2| + 1 - \bar{m} < |N_1| - \bar{m}$ if he or she chooses F, and

$$\hat{\omega}_U = (L1, \{1, 2\}, U, 0)$$

if he chooses U.

Given these experiences, the player in N_1 may come up with a model in which the payoff function, if F is chosen, is not given by $u(F, m)$ as specified above, but by

$$v(F, E) = \begin{cases} |N_1| & \text{if } E = \{1\}, \\ x & \text{if } E = \{1, 2\}. \end{cases}$$

The model constructed from the original festival game by replacing u by v is coherent with his or her experiences, and simpler than the original one if one uses a criterion under which the smaller the set of image of the payoff function is, the simpler is the model (given other things). In the sense that the mere presence of N_2 reduces the payoff value, v exhibits prejudices against N_2.

3.5 Pioneers

> "I think the importance of being a pioneer is that you have to be successful, ... Being successful leads to the next player, and the next player and so on."
>
> –Don Nomura, the agent of Hideo Nomo[13]

In 1995, Hideo Nomo, a Japanese pitcher, signed a contract with Los Angeles Dodgers after a contract dispute with Kintetsu Buffaloes. He was the second Japanese baseball player to make a Major League debut, only after the nearly forgotten Masanori Murakami. Nomo's games were regularly broadcast in Japan. Unlike Murakami, Nomo exceeded the expectations of the Japanese media and fans. His success inspired many baseball stars like Ichiro and Matsuzaka to come to the United States, too. Before Nomo, neither Japanese player nor club team had ever even dreamed of succeeding in Major League Baseball (MLB). Nobody ever predicted before 1995 that Japanese players could compete with MLB players. A transfer to MLB was not even in their scope. After 1995, a door to MLB unexpectedly opened to Japanese players all the sudden.

Pioneering works have one thing in common. All of them change the scope of people. In fact, this is almost the definition of a "pioneer." After having observed numerous instances of Japanese players' successes and failures with respect to playing in Japan, and, with respect to playing in the US, only one forgotten instance of failure and no success, it is not difficult to imagine that people construct a model, wherein playing in the US is not even an option.

4 Conclusion

We set forth a theory of man that tries to understand the world by constructing a model. We take experiences, or chunks of impressions, as primitives of the theory. A model is something that is constructed by agents. In doing so, agents use axioms such as coherence, according to which an agent can explain his own experiences, the behavior rule with respect to a solution concept, and "simplicity."

Inductive and deductive game theory should not be regarded as substitutes, rather, they can be viewed as complements. In reality, people use both induction and deduction in accumulating knowledge, which eventually affects their behavior. Further, the

[13] Quoted in the article "Wally Yonamine" by Rob Smaal in English edition of Asahi.com, Jan. 2, 2007.

construction of a model does not have to precede action taking. In fact, experiences include those impressions that are obtained through one's own behavior.

One may doubt how this theory might contribute to economics. One answer to this question is that we would better understand the nature of human behavior if we incorporate the kind of inductive inference presented here than otherwise. Ignoring an inductive inference in constructing a theory of man is similar to making a one-wing airplane. In particular, the idea of induction is required when we try to understand differences in viewpoints. For example, without an inductive view, we would not be able to understand the differences between the US and Japan with respect to their attitudes toward the market principle, which creates substantial differences concerning various issues ranging from labor practice to antitrust enforcement. These applications, however, are open to future research.

If we say a "model of the world," it may sound as though there existed an object called "the world." Although we do not know whether there exists a situation called an objective world or not, the concept of objective game itself is a creation of researchers. An "objective game" is constructed by a researcher in order to understand our experiences/impressions better than otherwise. However, in the present framework, we do not have to presume the existence of such an objective world, nor do we have to take a position against it. Without entering into such a metaphysical discourse, the present framework can be used to address issues that the current society confronts.

Different individuals create different worlds. This idea can be seen in *Vijñānavāda* (the doctrine of consciousness), a school of *Mahāyāna* (greater vehicle), founded by Asanga and Vasubandhu (5th century AD). They used the parable "*Issui-Shiken*," or "One water, four appearances": what humans view as "water" may be viewed as a "bloody sea" by *gaki* (hungry ghosts), as a "residence" by fish, and as a "land of treasure" by heavenly beings.

Our experiences are limited in various ways. We cannot feel what others feel. All we can do is to infer others' feelings from circumstances and their facial and other expressions. When we do it, we have already constructed a model of others. In this sense, it may well be the case that animals other than humans have some ability to construct a model. After all, God "created man in his own image" (Genesis 1:27) as believed in the West, while humans and animals transmigrate into each other in the East.

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