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On Understanding Experiences of Disability

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How do we grasp an experience as a disability? To this open question, we present in this article a schematic of an agent's cognition of taking an object as a being with an attribution, by formalizing its process. Under this model, the agent is able to be both aware of and unaware of other agent's disability according to her imagination and inference, and we emphasize the indefiniteness, creativity, and intersubjectivity of (the concept of) disability. Their implications for disability studies and economics are also mentioned.
Keywords: concept of disability, cognition of agent, intersubjectivity, channel theory, framework for disability studies and economics

1 Introduction

Grasping an experience as a disability: it is this simple cognition process that we explore in this article. Stated it more generally, we explore the ability to cognize an object as a being with an attribution. We could say that this is an ability which we presuppose (or have to presuppose) as obvious when we cognize the world. At the same time, it is undeniable that it is an ability with a certain kind of indefiniteness. In fact, the judgement of what experience can (or should) be deemed a disability varies according to the context and situation, or even the person who judges it. The aim of this article is to clarify what creates this indefiniteness and to highlight a creative aspect of cognition by formalizing the schematic of this ability.

We will begin by presenting an example which the authors actually experienced in order to specify this problem more concretely. It was an ordinary event, but forced us to rethink our understanding of the concept of 'disability'.

S was waiting for an elevator.[1] N, a research colleague who attends the same research forum, was also there. A minute later, the elevator stopped on our floor. Then the door opened and N asked S a question, looking at the mirror set up in the elevator.

— Can you imagine what really troubles us when it's missing from a lift?

— Ummm ...

— A mirror. It's crucial for us.

Some explanation is needed to interpret this example. N requires a wheelchair for everyday life. As is well known, he needs the mirror to confirm the safety of the rear side of his wheelchair, particularly when getting off the lift.

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N would have been made conscious of this 'disability' whenever he rode in a lift. Anybody who has ever rode in a lift with a wheelchair must be conscious of this. However, S was not aware of this obvious fact at that time.

This made S ponder. There are experiences that we can be unaware of being a disability but also that we can be aware of and cognize it as a disability. What makes this experience possible or what happened in this experience?

Let us first confirm that S did not change his syntactic structure of 'disability' from this experience. He understands the concept of 'disability' as 'an experience which is brought about by a certain type of barrier'. It was a change of the correspondence between 'disability' and its referent that S went through from that event, i.e., 'the barrier of getting off a lift without a mirror in a wheelchair' is a 'disability'.

It is more important that S did not experience the 'barrier' directly himself. S was only made aware by N of a troublesome situation that he had ever seen. In other words, S changed the correspondence between his concept of 'disability' and its referent only due to the indication that the 'experience' which he had witnessed was a 'barrier' (and by exercising his imagination). Therefore, this change must have been brought about by N's utterance alone as a momentum.

We present in this article a model of the schematic which makes this change possible. The model captures at least one aspect of our cognition, that agents can be both unaware of some experience being a disability and, at the same time, be aware of it as a disability, and highlights the function of the concept of 'disability' as a momentum which sets off this change.

We have the ability to cognize an object as a being with an attribution. To question this ability afresh may seem meaningless as it is too obvious. However, this is an important question not only for disability studies but for economics. For example, many of the anomalies of human behavior which behavioral economics has reported, such as the framing effect, could be essentially elucidated by the change of agents' cognition. Problems such as nonmonotonic reasoning in logic and A.I. are similar. When we consider the change of cognition in these problems, deliberations on the question 'what do we deem a disability' can give thought-provoking suggestions especially for economics, which has made light of agents' cognition of themselves.

The rest of this article consists of four sections. In Section 2, we introduce fundamental concepts necessary for the framework by which we formalize the argument. In Section 3, we set up a formal framework and give a definition of 'understandability' which indicates that 'one can understand that an experience can be grasped as a disability'. In Section 4, we give an formal expression to the example in this section. Section 5 concludes this article.

2 Preliminaries

In this section we introduce the basic concepts constituting the framework of channel theory. Channel theory is a formal framework established by Barwise & Seligman (1997) to model information flows. We apply it to model an agent's cognition.

2.1 Classifications

One of the most fundamental concepts of channel theory is *classification*. First we define it as follows.

Definition 1 (Barwise & Seligman (1997))**.** A *classification* $A = \langle \mathrm{tok}(A), \mathrm{typ}(A), \vDash_A \rangle$ consists of

1. a set, $\mathrm{tok}(A)$, of objects to be classified, called the *tokens of* A,
2. a set, $\mathrm{typ}(A)$, of objects used to classify the tokens, called *type* of A, and
3. a binary relation, $\vDash_A$, between $\mathrm{tok}(A)$ and $\mathrm{typ}(A)$.

If $a \vDash_A \alpha$, then a is said to be *of type* α *in* A.

Below we describe an agent's *observation* as a classification, i.e., the tokens are the subjects of the observation and the types are the observed attributions.

2.2 Infomorphisms and channels

Next we introduce *infomorphism* which detects a certain kind of sameness of classifications.

Definition 2 (Barwise & Seligman (1997))**.** An *infomorphism* $f : A \rightleftarrows B$ from A to B is a contravariant pair of functions $f = \langle f^{\wedge}, f^{\vee} \rangle$ satisfying the following *Fundamental Property of Infomorphisms*:

$$f^{\vee}(b) \vDash_A \alpha \quad \text{iff} \quad b \vDash_B f^{\wedge}(\alpha)$$

for each token $b \in \mathrm{tok}(B)$ and each type $\alpha \in \mathrm{typ}(A)$. Classification A is called the *domain* of f and classification B is called the *codomain* of f.

If there exists an infomorphism f between two classifications, we say that the classifications have the same structure in the sense of the infomorphism f. If a family of infomorphisms have a classification as a common codomain, the family of infomorphisms is called a *channel* and the classification a *core*. The definitions are as follows.

Definition 3 (Barwise & Seligman (1997))**.** A *channel* $\mathcal{C}$ is an indexed family $\{f_i : A_i \rightleftarrows C\}_{i \in I}$ of infomorphisms with a common codomain C, called the *core* of $\mathcal{C}$. The tokens of C are called *connections*; a connection c is said to *connect* the tokens $f_i(c)$ for $i \in I$. A channel with index set $\{0, \ldots, n-1\}$ is called an *n-ary* channel.

In this article, we describe an agent's cognition as a binary channel. The channel describes the function of making a connection between the left-hand-side classification (called the *source*) and the right-hand-side classification (called the *target*), and we utilize this feature of the channel to model the agent's cognitions.

2.3 Theories

Next we bring up a concept of *theory* to model the agent's knowledge and inference process. We assume in this article that all agents make an inference according only to the knowledge expressed by this theory.

Definition 4 (Barwise & Seligman (1997))**.** Let Σ be an arbitrary set. A binary relation $\vdash$ between the subsets of Σ is called a *(Gentzen) consequence relation on* Σ. A *sequent* is a pair $\langle \Gamma, \Delta \rangle$ of subsets of Σ and a sequent is called a *partition* of a set Σ' if $\Gamma \cup \Delta = \Sigma'$ and $\Gamma \cap \Delta = \emptyset$. A *theory* is a pair $T = \langle \Sigma, \vdash_T \rangle$, where $\vdash_T$ is a consequence relation on Σ of theory T. A *constraint* of the theory T is a sequent $\langle \Gamma, \Delta \rangle$ of Σ for which, $\Gamma \vdash_T \Delta$. A sequent $\langle \Gamma, \Delta \rangle$ is *T-consistent* if $\Gamma \nvdash_T \Delta$.

We assume regularity below as a property of the theory.

Definition 5 (Barwise & Seligman (1997))**.** A *theory* $\mathrm{T} = \langle \Sigma, \vdash_T \rangle$ is *regular* if it satisfies the following for all types and all sets $\Gamma, \Gamma', \Delta, \Delta', \Sigma', \Sigma_0, \Sigma_1$ of types:

Identity: $\alpha \vdash_T \alpha$.

Weakening: If $\Gamma \vdash_T \Delta$, then $\Gamma, \Gamma'' \vdash_T \Delta, \Delta'$.[2]

Global Cut: If $\Gamma, \Sigma_0 \vdash_T \Delta, \Sigma_1$ for each partition $\langle \Sigma_0, \Sigma_1 \rangle$ of Σ', then $\Gamma \vdash_T \Delta$.

Furthermore, when an arbitrary theory T is given, we define the *regular closure* of theory T as the theory which includes all the constraints derived from the constraints of theory T by the regular operations above.

Definition 6 (Barwise & Seligman (1997), Sakahara & Sato (2008))**.** Let arbitrary theory $T = \langle \Sigma, \vdash_T \rangle$ be given. The smallest theory whose constraints are comprised of the constraints of theory T and all the constraints derived from the constraints of theory T is called the *regular closure* of theory T and is denoted as $\bar{T} = \langle \Sigma, \vdash_{\bar{T}} \rangle$.

The existence of the regular closure is guaranteed by Proposition 9.7. of Barwise & Seligman (1997).

2.4 Cognizance classification

Finally, we give a definition of a particular classification which will appear in this article. As previously noted, we utilize the concept of channel to describe an agent's cognition, and give priority to the core of the channel as a mirror of the agent's cognition. We use as the core a classification which is defined by the structure of source and target, and, moreover, reflects the agent's own knowledge. This classification is called *cognizance classification*.

The definition of cognizance classification is given in Sakahara & Sato (2008) in a rigorous manner. However, we simplify it here since we can ignore some problems to be considered in that definition.

We define cognizance classification in the following manner. First we define a set of *possible* and *realizable states* which is the set of tokens of the core.

Definition 7 (Barwise (1997), Sakahara & Sato (2008))**.** Let A be a source, B be a target classification, and T be a regular theory. Firstly, the set of all partitions of $\mathrm{typ}(A) \cup \mathrm{typ}(B)$ is said to be the set of *states* generated by A and B, written $\Omega_{\langle A,B\rangle}$. Secondly, the set

$$\Omega^{R}_{\langle A,B\rangle} = \{\langle \Theta, \Lambda\rangle \in \Omega_{\langle A,B\rangle} \mid \exists a \in A, \quad \mathrm{typ}(a) \subseteq \Theta \ \text{ and } \ \mathrm{typ}^{c}(a) \subseteq \Lambda\}$$

is said to be the set of *realizable states* generated by A and B under A. Thirdly, the set

$$\Omega^{IP}_{\langle A,B\,|\,T\rangle} = \{\langle \Theta, \Lambda\rangle \in \Omega_{\langle A,B\rangle} \,|\, \langle \Theta, \Lambda\rangle \in \vdash_{T}\}$$

is said to be the set of *impossible states* under the theory T. Fourthly, the set

$$\Omega^{P}_{\langle A,B\,|\,T\rangle} = \Omega_{\langle A,B\rangle} \setminus \Omega^{IP}_{\langle A,B\,|\,T\rangle}$$

is said to be the set of *possible states* under the theory T, which are not impossible under T. Finally, the set

$$\Omega^{PR}_{\langle A,B\,|\,T\rangle} = \Omega^{P}_{\langle A,B\,|\,T\rangle} \cap \Omega^{R}_{\langle A,B\rangle}$$

is said to be the set of *possible and realizable states* under the theory T and the tokens of A.

Then we define a cognizance classification whose tokens are the possible and realizable states under the theory T.

Definition 8. Given a source A, a target B, and a regular theory T, the *cognizance classification* $C_{\langle A,B,T\rangle}$ is the classification generated by A, B, and T such that:

$$C_{\langle A,B,T\rangle} = \left\langle \Omega^{PR}_{\langle A,B\,|\,T\rangle},\ \mathrm{typ}(A) \cup \mathrm{typ}(B),\ \vDash_{C_{\langle A,B,T\rangle}} \right\rangle$$

where $\vDash_{C_{\langle A,B,T\rangle}}$ is defined as:

$$\langle \Theta, \Lambda\rangle \vDash_{C_{\langle A,B,T\rangle}} \alpha \qquad \text{iff} \qquad \alpha \in \Theta.$$

3 Formulation

In this section we give a formal expression of the function of cognition under which an agent deems an experience to be a 'disability'. By this formulation, we can highlight the creativity of cognition and make a function of the concept of 'disability' out to be a momentum.

First we consider a society $I = \{1, \ldots, N\}$ consisting of N agents. Then picking an *observer* agent $i \in I$, we investigate observations from the viewpoint of i.

The observer i observes certain experiences $\{e_1, \ldots\}$. We denote this observation O^{E}. The token observed is an intentional object common to all agents and the type observed consists of Db and Bar, with the former indicating disability and the latter barrier.[3]

At the same time, i makes an observation of the society I. We denote this observation O^I. The token observed is I. The type observed consists of Chr and $\langle e_m, \alpha \rangle$ where $\alpha \in \{\mathrm{Db}, \mathrm{Bar}\}$. The former Chr indicates that the agent whose experience is observed by i has physical (we mean by this word both material and bodily) characteristics and the latter $\langle e_m, \alpha \rangle$ indicates how each agent whose experience is observed deems her own experience.

We call the latter, which is detected by direct utterance or appearance and is independent of the agent's own observation on her experience, *intersubjective* type. For example, the type $\langle e_m, \mathrm{Bar} \rangle$ indicates that the experience e_m is a barrier and the type $\langle e_m, \mathrm{Db} \rangle$ that the experience e_m is a disability.

Moreover, we assume the relation below between observations on i's own experience and on society.

$$e_m \models_{O^E} \alpha \qquad \text{iff} \qquad i \models_{O^I} \langle e_m, \alpha \rangle.$$

In other words, observations on one's own experience and on society are consistent. This is guaranteed by letting $\models_{O^E} = \mathrm{typ}_{O^I}(i)$ where $\mathrm{typ}_{O^I}(i) \equiv \{\langle e_m, \alpha \rangle \in \mathrm{typ}(O^I) \mid \langle i, \langle e_m, \alpha \rangle \rangle \in \models_{O^I}\}$.

Next we formulate observer i's knowledge on the types defined above. We define that i deems an experience e_m to be a disability Db if and only if (1) e_m is experienced by agents with physical characteristics Chr, and (2) e_m relates (or seems to relate) the environmental or institutional barrier Bar. We can express the fact that i's knowledge includes the type $\langle e_m, \mathrm{Db} \rangle$ as the following constraints:

$$\underset{(1)}{\mathrm{Chr}}, \quad \underset{(2)}{\langle e_m, \mathrm{Bar} \rangle} \quad \vdash_K \quad \langle e_m, \mathrm{Db} \rangle,$$

and

$$\langle e_m, \mathrm{Db} \rangle \vdash_K \mathrm{Chr}, \tag{1}$$

$$\langle e_m, \mathrm{Db} \rangle \vdash_K \langle e_m, \mathrm{Bar} \rangle. \tag{2}$$

The first constraint can be interpreted as a necessary condition and the remaining two as sufficient conditions. We assume $\mathrm{typ}(O^E) \cup \mathrm{typ}(O^I) \subseteq \mathrm{typ}(K)$, i.e., the knowledge K includes the types of observation both on i's experience and on society.

Finally, under the above settings, we define an agent i as being able to understand an experience of disability of an agent $j \in I$ as follows.

Definition 9. Suppose an agent $j \in I$ with a physical characteristic Chr deems her own experience e to be a disability Db, i.e., $j \models_{O^I} \mathrm{Chr}$ and $j \models_{O^I} \langle e, \mathrm{Db} \rangle$. Then i *can understand* that j deems her experience e to be a disability if the token of which type is only Chr is connected only to the token of which type is $\langle e, \mathrm{Db} \rangle$ under a channel with the cognizance classification with respect to i's knowledge as a core.

In other words, if i can imagine a barrier with j's experience by a physical characteristic and take it as a disability according to i's knowledge, we say that i can understand that j deems her experience to be a 'disability'. We call this condition 'understandability (of disability)' hereafter.

4 The example

In this section, based on the settings in the previous section, we give a formal expression to the example which we presented at the beginning. Specifically, we consider a society consisting of $I = \{S, N\}$ with the observer S. We also consider only one token y, an experience of 'getting on a lift without a mirror', which is the intentional object common to S and N.[4] Moreover, we set Chr as the type of a specific physical characteristic 'needing a wheelchair'. As knowledge, S only has the constraints presented in the previous section.

4.1 How we fail to understand experiences of disability

Let us first examine the concept of 'disability' which S had before the conversation with N. S did not know either the barrier or disability caused by the experience of 'getting on a lift' both for himself and for N until the conversation. It could be said that he was not aware of the types. By contrast, S had observed that N was a wheelchair user. Therefore, we can express S's observation of the society O_0^I with the following classification table.

Table 1: Classification table of S's observation O_0^I on the society I

$\models_{O_0^I}$	Chr
N	1
S	0

Then, S was asked by N what was troublesome about getting on a lift with a wheelchair. However, S could not answer the question properly, and it means he did not understand that 'needing a wheelchair' could have a relation with the experience of getting on a lift. In other words, according to the condition of understandability presented in the previous section, we could say that S could not understand that N deemed the experience of getting on a lift without a mirror to be a disability even though S imagined a situation in which he needed a wheelchair. So we next consider what is it that one does not understand about the disability which someone else experiences, according to the definition of understandability.

Here, in order to model S's cognition, we consider the observation $O^{I'}$ of an imaginary society I' in which the observer S himself 'needs a wheelchair'. That is, we verify whether we can imagine that a certain experience is a barrier and a disability or not by imagining a society in which we ourselves 'need wheelchairs'. We first consider a classification B based on an image of a society comprised of X and Y, the former being an agent who deems a lift without a mirror to be a barrier and the latter an agent who does not. The classification tables of $O^{I'}$ and B are in Table 2.

That is, considering a channel of which the source is the observation $O^{I'}$ and the target is the observation B, we can see the relationship with respect to S's cognition

Table 2: Classification tables of imaginary observation $O^{I'}$ and image B

$\models_{O^{I'}}$	Chr
S'	1

$\models_B$	$\langle y, \mathrm{Bar} \rangle$
X	1
Y	0

between an agent who needs a wheelchair and an agent who perceives an experience of getting on a lift without a mirror as a disability.

In order to construct this channel, we compose a cognizance classification $C_0 = C_{\langle O^{I'}, B, \bar{K} \rangle}$ as a core of this channel using observer S's own knowledge K. In accordance with definition 7 and definition 8, we can derive this cognizance classification C_0 as in Table 3.[5]

Table 3: Classification table of cognizance classification $C_0 = C_{\langle O^{I'}, B, \bar{K} \rangle}$

$\models_{C_0}$	Chr	$\langle y, \mathrm{Bar} \rangle$
X	1	1
Y	1	0

Adopting this classification C_0 as a core, we can construct a channel $\mathcal{C}_0 = \langle f_0, g_0 \rangle$ between the classification $O^{I'}$, which is an observation on the imaginary society, and B, an image of the imaginary society.

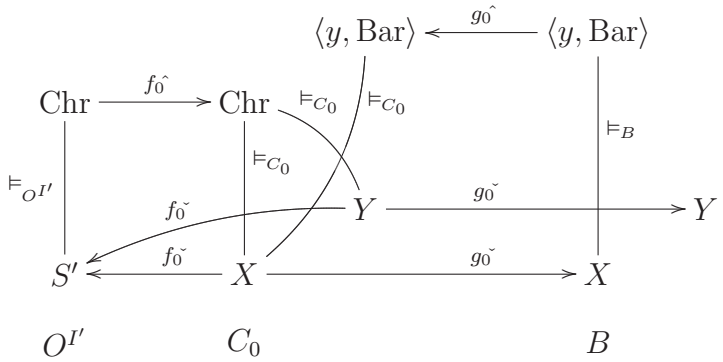

Figure 1: Channel $\mathcal{C}_0$ and its core C_0

We find that S' is connected to X, who deems a lift without a mirror to be a barrier, via the channel $\mathcal{C}_0$. The token of the core X connecting S' and X verifies this. However, S' is also connected to Y, another token of B. That is, S' is also connected to an agent who does not deem a lift without a mirror to be a barrier.

There are connections of S' not only to X but also to Y. We interpret this as a situation where S has no way of telling whether he should 'deem a lift without

a mirror to be a barrier or not'. We mean that S' is connected to both X and Y because S cannot detect any relation of consequence between 'needing a wheelchair' and 'perceiving a lift without a mirror as a barrier'.

Investigating S's knowledge K supports the reasonableness of our interpretation. The knowledge K only includes constraints related to the definition of 'disability' as previously noted. On the contrary, the knowledge K does not include any relations of consequence between 'needing a wheelchair' and 'perceiving a lift without a mirror as a barrier' as constraints. We can interpret this as a situation in which S has no proper knowledge to distinguish these types. And it could be said that the lack of proper knowledge between these types causes S to connect S' to X, who 'deems a lift without a mirror to be a barrier', and Y, who 'does not'.

By the above we have confirmed that a lack of proper knowledge can be a restraint of understanding disability. That is, there are cases where we cannot understand a disability which an other agent experiences if we cannot imagine the relationship between 'needing a wheelchair' and 'perceiving a lift without a mirror'.

4.2 How we do understand experiences of disability

In this subsection we consider S's observation of the society after hearing N's words that getting on a lift without a mirror is troublesome. At this moment, S can observe that N deems the token y to be a barrier. So we can express S's observation O_1^I of the society as follows.

Table 4: Classification table of S's observation O_1^I of the society I

$\models_{O_1^I}$	Chr	$\langle y, \mathrm{Bar} \rangle$
N	1	1
S	0	0

This classification O_1^I is more informative than O_0^I since it reflects the cognition of N who deems 'getting on a lift without a mirror to be a barrier'. However, we cannot conclude that S can only understand the 'disability' of N by this classification, because it does not indicate directly that N perceives the experience as a 'disability', or, even though N himself mentioned it, it does not necessarily make S understand N's experience as a 'disability'. In fact, this classification does not show that S deems 'getting on a lift without a mirror' to be a barrier, and this reflects the fact that S does not understand why the token y could be a disability. Thus we next consider what momentum makes S come to understand N's cognition.

Let us recall what N said. N did not simply state to S that 'getting on a lift without a mirror is a barrier'. N asked S 'When we get on an elevator, can you imagine what troubles us the most when it's missing?' That is, N asked S to imagine factors that might act as a barrier to moving his wheelchair beforehand,

and gave the answer 'a mirror'. According to this guidance, S received a hint for understanding why 'a lift without a mirror' could be a barrier.

At this time S came to the following realization. Since it is often difficult to turn around the wheelchair in the lift, it is also difficult to confirm the safety of the rear side of the wheelchair. However, in most cases, the doorway is located backward if one entered the lift moving forward. Therefore, one cannot confirm the safety of the rear side without a mirror. Thus, being without a mirror can be a barrier to getting off the lift safely.

We can simplify this idea and inference process and, for example, express it by a proof figure consisting of the following two constraints.

$$\mathrm{Chr} \vdash_{Ab} \langle y, \alpha \rangle \qquad \text{and} \qquad \langle y, \alpha \rangle \vdash_{Ab} \langle y, \mathrm{Bar} \rangle,$$

where α indicates 'cannot view the rear side'. Then we can interpret the first constraint as 'if one needs a wheelchair, then one cannot view the rear side in a lift without a mirror' and the second as 'if one cannot view the rear side in a lift without a mirror, then the lift is a barrier'. These constraints enable us to infer that 'if one needs a wheelchair, then a lift without a mirror is a barrier'. The following proof figure depicts this inference process.

$$\dfrac{\dfrac{\mathrm{Chr} \vdash_{\bar{A}b} \langle y, \alpha \rangle}{\mathrm{Chr} \vdash_{\bar{A}b} \langle y, \alpha \rangle, \langle y, \mathrm{Bar} \rangle}\ (\text{Weakening}) \qquad \dfrac{\langle y, \alpha \rangle \vdash_{\bar{A}b} \langle y, \mathrm{Bar} \rangle}{\mathrm{Chr}, \langle y, \alpha \rangle \vdash_{\bar{A}b} \langle y, \mathrm{Bar} \rangle}\ (\text{Weakening})}{\mathrm{Chr} \vdash_{\bar{A}b} \langle y, \mathrm{Bar} \rangle}\ (\text{Global Cut})$$

It is essential that S needs not to know that 'if one needs a wheelchair then a lift without a mirror is a barrier'. What is important is that one can get this conclusion by inference, from two constraints which are relatively easy to keep in mind. We often draw a conclusion which is difficult to attain immediately by accumulating simple ideas and inferences. Such inferences made it possible for S to imagine that 'a lift without a mirror is a barrier' by seeing himself as having N's physical characteristic 'needing a wheelchair'.

Let these two constraints add to the knowledge K, and let it be written $K' = \langle \mathrm{typ}(K) \cup \mathrm{typ}(Ab), \vdash_K \cup \vdash_{Ab} \rangle$ where Ab is a theory consisting of only the two constraints above. Then, by using the revised knowledge K', we can compose a cognizance classification $C_1 = C_{\langle O^{I'}, B, \bar{K}' \rangle}$ between two images of society previously discussed $O^{I'}$ and B.

Table 5: Classification table of cognizance classification $C_1 = C_{\langle O^{I'}, B, \bar{K}' \rangle}$

$\vDash_{C_1}$	Chr	$\langle y, \mathrm{Bar} \rangle$
X	1	1

By letting this classification C_1 be a core, we can construct a channel $\mathcal{C}_1 = \langle f_1, g_1 \rangle$ between $O^{I'}$ and B, as shown in Figure 2.

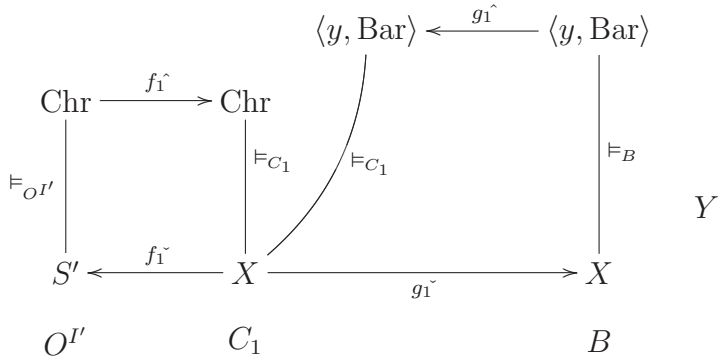

Figure 2: Channel $\mathcal{C}_1$ and its core C_1

One can see that S can perceive the lift without a mirror as a barrier though he himself cannot experience it directly under the channel $\mathcal{C}_1$. We can verify this by S' not being connected to Y, which was connected in $\mathcal{C}_0$ as previously discussed. Moreover, S can also understand, in the light of the definition of understandability, that the experience y can be deemed a disability. We will check this next.

Here we consider a classification D which describes a society consisting of R and S, the former being an agent who deems the experience y of getting on a lift without a mirror to be a disability and the latter being one who does not. Then we investigate a channel where the source is $O^{I'}$ and the target D. We can depict D as in the following classification table.

Table 6: Classification table of classification D

$\vDash_D$	$\langle y, \mathrm{Db}\rangle$
R	1
S	0

By using the definition of disability which the knowledge K originally includes, we can compose a cognizance classification $C_2 = C_{\langle C_1, D, \bar{K}'\rangle}$ between C_1 and D as follows.

Table 7: Classification table of cognizance classification $C_2 = C_{\langle C_1, D, \bar{K}'\rangle}$

$\vDash_{C_2}$	Chr	$\langle y, \mathrm{Bar}\rangle$	$\langle y, \mathrm{Db}\rangle$
R	1	1	1

By way of C_1, and letting C_2 be a core, we can construct a channel $\mathcal{C}_2 = \langle m_2, \ell_2\rangle$ in which $m_2 = h_2 \circ f_1$. This channel is shown in Figure 3.

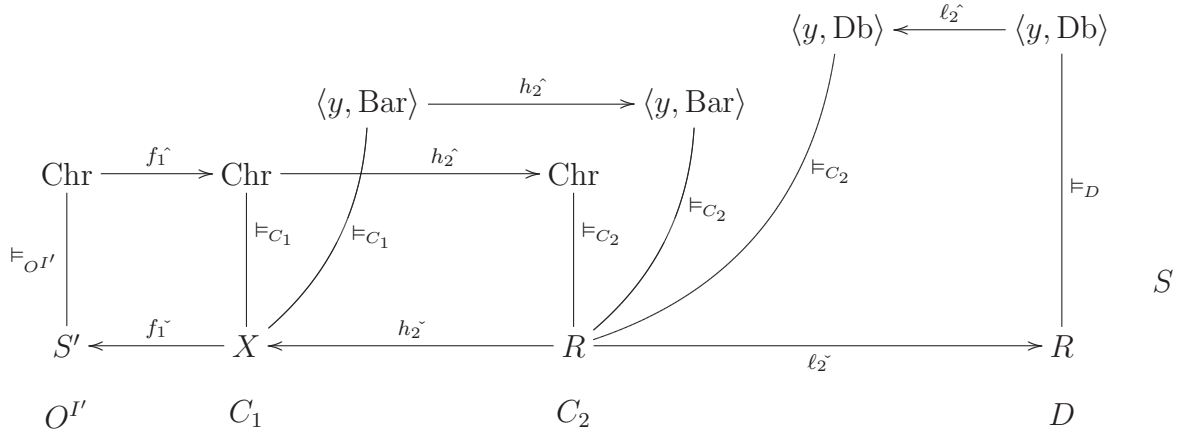

Figure 3: Channel $\mathcal{C}_2$ and its core C_2

Under this channel $\mathcal{C}_2$, S can come to understand a 'disability', which he himself cannot experience directly, involved in getting on a lift without a mirror. And S does not connect 'needing a wheelchair' to 'not perceiving the experience of getting on a lift without a mirror as a disability'. Thus we can say that S understands N's disability in the light of the definition of understandability.

5 Concluding remarks

In closing, we investigate the functions of knowledge and imagination, one aspect of which we looked at in the previous section.

The example in the previous section can be summerized as follows. At first, S did not understand the disability which N experienced. This was because S lacked knowledge about the relation of consequence between 'needing a wheelchair' and the 'barrier experienced by getting on a lift without a mirror'. This inability to understand was resolved by S imagining a link between them by himself. In other words, he came to infer a relation of consequence between them by imagining two constraints, 'if one needs a wheelchair then one cannot view the rear side in a lift without a mirror', and 'if one cannot view the rear side in a lift without a mirror then the lift is a barrier'.

It is noteworthy that it is not necessary to confine the constraints leading S to deem N's experience to be a disability to the ones outlined above. They are not the only way in which N can come to deem his experience to be a disability, too. The basis and validity of S's imagination does not go beyond the fact that its consequence is consistent with S's experience and N's utterance. In fact, many of our imagining in everyday life takes place in a similar way. It is not claiming objective validity that we do when we take an experience to be a disability. Rather, it is the expression of subjective or intersubjective understanding which makes us highlight the barrier which is brought to light in the experience of the agent, and interpret a friction with

the environment or society in a comprehensible way for ourselves. And since this way of understanding is defined by our relationship with the partners, the environment, or society, it becomes a kind of judgement of which way of understanding defines the relationship between the interpreter and the partner, environment or society. So we can see that, by imagining something and making a judgement, S creates the society itself, in which he appears as an agent, made up between N, S himself and the reader reading this text.

This leads to two implications. One implication is for disability studies. One of the most important role of disability studies is to give people like S, who do not experience and are not aware of disabilities, proper knowledge and to get them to understand disabilities by accumulating simple imaginatings and inferences. By doing that, making disability into an intersubjective existence we can create such a world.

Another implication is for economics. When economists attempt to analyze society objectively, they do so using models in which everything is defined by the economists themselves. How an agent's cognition functions is not defined, because it cannot be defined 'objectively'. However, in order to analyze disability, which exists 'intersubjectively', a framework which can formally handle the cognition of an agent who interprets and, at the same time, creates the world, is essential. We believe the approach presented in this paper has the potential to do that.

Notes

1. Although S actually refers to both of the authors of this article, we treat it as singular, masculine noun for descriptive convenience. N is also masculine.

2. A constraint $\Gamma, \Gamma'' \vdash_T \Delta, \Delta'$ should principally be written as $\Gamma \cup \Gamma'' \vdash_T \Delta \cup \Delta'$. $\alpha \vdash_T \alpha$ should also be written as $\{\alpha\} \vdash_T \{\alpha\}$. However, for notational convenience, we simplify the expression of constraint according to custom. Hereafter, we will describe $\{\alpha, \beta\} \vdash \{\gamma\}$ as $\alpha, \beta \vdash \gamma$ and $\Gamma \cup \Gamma' \vdash \Delta \cup \{\alpha\}$ as $\Gamma, \Gamma' \vdash \Delta, \alpha$,too.

3. Although the identity of the token should be checked by observations of society, we presume it to avoid vexation.

4. Strictly speaking, this means that there is an observation of society below. Here, M indicates the type which corresponds to the experience of 'getting on a lift with a mirror'.

$\models_{O^I}$	$\langle y, \mathrm{M} \rangle$
N	0
S	0

5. It is easy to compose the cognizance classification C_0 from the knowledge K. Since the regular closure $\bar{K}$ of the knowledge K does not include any partitions generated by Chr or $\langle y, \mathrm{Bar} \rangle$, it holds that both $\mathrm{Chr} \nvdash_{\bar{K}} \langle y, \mathrm{Bar} \rangle$ and $\mathrm{Chr}, \langle y, \mathrm{Bar} \rangle \nvdash_{\bar{K}}$. Thus $\Omega^{PR}_{\langle O^{S'}, B \mid \bar{K} \rangle}$ includes only the counterexamples X and Y to these constraints.

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