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Abstract

This paper considers social choice theory without the Pareto principle. We revisit the trade-off theorem developed by Campbell and Kelly (Econometrica 61:1355–1365, 1993), and generalize their result. By introducing an alternative criterion of power structure, a dominance relation, we show that if a social welfare function on the unrestricted domain satisfies the independence of irrelevant alternatives, then it must be exist either maldistribution of power or too many pairs of alternatives which social ranking is fixed independently of individual preferences. Moreover, we offer two applications of our main result.

JEL classification D61; D71

Keywords Social welfare function; Trade-off theorem; Independence of irrelevant alternatives; dominance relation.

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1 Introduction

The Pareto principle is a central normative criterion in welfare economics and social choice theory. However, it has "independence" property and thus it causes the "Paretian epidemic" (Sen, 1976). One of the most well-known epidemic is the so-called *field expansion lemma*: if a Paretian social welfare function satisfies independence of irrelevant alternatives, then an individual must be a dictator whenever he/she is decisive for some pairs. This lemma is known as the main cause of Arrow's impossibility theorem. A route for escape from the impossibility is to drop the Pareto principle, and many researchers have studied social choice theory without Pareto.[1]

The seminal work of Campbell and Kelly (1993, 1997) considers how much mileage can be got out of the impossibility by relaxing the Pareto principle. According to them, if a social welfare function satisfies independence of irrelevant alternatives (the Pareto principle is dropped), at least half pairs of alternatives is fixed independently of individual preferences whenever there exists no individual who is decisive over at least half pairs. Their result is known as the *trade-off theorem*. Since trade-offs are central problems of economics, Campbell and Kelly's work gives a new perspective of social choice theory.[2]

The purpose of this paper is to re-examine Campbell and Kelly's trade-off theorem and to provide further remarks: we specify an alternative criterion of power structure and generalize their result. It is shown that if a social welfare function satisfies independence of irrelevant alternatives, then it must be exist either maldistribution of power or too many pairs of alternatives which social ranking is independently of individual preferences. To prove this result, we introduce the dominance relation, which represents an unequal distribution of power of a social welfare function. Since the dominance relation is operationally tractable, our general result is applicable to analysis of more specified power structure. We first obtain a result of Campbell and Kelly (1993) as a corollary to our theorem, and subsequently provide other applications of our theorem.

The rest of this paper is organized as follows. Section 2 introduces our notation and definitions. We present our main results in Section 3, and offer two applications of them in Section 4. Section 5 concludes this paper.

2 Preliminaries

The set of finite alternatives is X with $|X| = m \geq 3$.[3] Let $R \subseteq X \times X$ be a binary relation on X. It is customary to write, for all $x, y \in X$, xRy if and only if $(x, y) \in R$. Then, xRy means "x is at least as good as y". The symmetric and the asymmetric part of R are denoted by I and P, respectively.[4] We introduce the properties for a binary relation R defined on X. A binary relation R is (i) complete if and only of for all $x, y \in X$, xRy or yRx, (ii) transitive if and only if for all $x, y, z \in X$, $[xRy \text{ and } yRz] \Rightarrow xRz$. An ordering R on X is a complete and transitive binary relation on X. Let $\mathcal{R}$ be the set of all logically possible orderings R defined on X.

[1]See, among others, Murakami (1968), Wilson (1972), Fountain and Suzumura (1982), and Kelsey (1985). Miller (2009) is a recent work on this subject. See also Campbell and Kelly (2003).

[2]See Campbell and Kelly (1997) for a survey.

[3]For a set A, $|A|$ denotes its cardinality.

[4]That is, the two binary relations I and P are defined by $xIy \Leftrightarrow (xRy \text{ and } yRx)$ and $xPy \Leftrightarrow (xRy \text{ and } \neg yRx)$, respectively.

Let $N := \{1, 2, \ldots, n\}$ with $n \geq 2$ is a finite set of individuals. Each individual $i \in N$ has a preference ordering $R_i \in \mathcal{R}$ on X. A profile $\mathbf{R} = (R_1, R_2, \cdots, R_n) \in \mathcal{R}^n$ is an n-tuple of individual preference orderings on X. The restriction of preference profile $\mathbf{R}$ on X to the subset Y of X is denoted by $\mathbf{R}|Y$. A **social welfare function (SWF)** is a mapping $f : \mathcal{R}^n \to \mathcal{R}$ associating a social preference ordering $f(\mathbf{R}) \in \mathcal{R}$ with each profile $\mathbf{R} \in \mathcal{R}^n$. For simplicity, $f(\mathbf{R})$ (resp. $f(\mathbf{R}')$) is denoted as R (resp. R').

Let $Y \subseteq X$. An SWF f is **directly dictatorial** on Y if there exists $i \in N$ such that for all $\mathbf{R} \in \mathcal{R}^n$, for all $x, y \in Y, xP_iy \Rightarrow xPy$. An SWF f is **inversely dictatorial** on Y if there exists $i \in N$ such that for all $\mathbf{R} \in \mathcal{R}^n$, for all $x, y \in Y, xP_iy \Rightarrow yPx$. We say that f is **dictatorial** on Y (in short, $f|Y$ is dictatorial) if it is either directly dictatorial or inversely dictatorial on Y. An SWF f is **null** on Y (in short, $f|Y$ is null) if xIy for all $x, y \in Y$ and all $\mathbf{R} \in \mathcal{R}^n$. The ordered pair (x, y) is **fixed** by f if xIy for all $\mathbf{R} \in \mathcal{R}^n$ ($f|\{x, y\}$ is null), xPy for all $\mathbf{R} \in \mathcal{R}^n$, or yPx for all $\mathbf{R} \in \mathcal{R}^n$.

We next introduce a well-known axiom on an SWF f.

Independence of Irrelevant Alternatives: $\forall x, y \in X$, $\forall \mathbf{R}, \mathbf{R}' \in \mathcal{R}$, if $[xR_iy \Leftrightarrow xR'_iy]$ for all $i \in N$, then $[xRy \Leftrightarrow xR'y]$.

We call an SWF f satisfying this axiom **Arrovian**. The following lemma, shown by Wilson (1972), is the fundamental result on social choice without the Pareto principle.[5]

Lemma 1 (Wilson's Partition Theorem)
For every Arrovian SWF f, there exists a unique partition $\mathcal{Y} = \{Y_1, \cdots Y_k\}$ of X and a linear order $\succ$ on $\mathcal{Y}$ such that the following statements hold for each distinct $Y_i, Y_j \in \mathcal{Y}$:
(i) $f|Y_i$ is dictatorial, or null, or Y_i contains exactly two members;
(ii) $Y_i \succ Y_j$ if and only if $\forall x \in Y_i, \forall y \in Y_j, \forall \mathbf{R} \in \mathcal{R}^n, xPy$.

The pair $(\mathcal{Y}, \succ)$ in Lemma 1 is called a **component structure** induced by f. We will, for convenience, call $\mathcal{Y}$ the component structure induced by f when we need not to refer to $\succ$. We refer to each $Y_i \in \mathcal{Y}$ as a **component** of f, and denote $m_i := |Y_i|$.

Note that Wilson's partition theorem establishes that (x, y) is fixed if x and y belong to different components or if x and y are distinct members of some null component. The only other possibilities are that $f|\{x, y\}$ is dictatorial or $\{x, y\}$ itself constitutes a component. An Arrovian SWF is said to be **locally dictatorial** if its restriction on each component is dictatorial, that is, $f|Y_i$ is dictatorial for each $Y_i \in \mathcal{Y}$.

We analyze the relation between component structures and the number of fixed pairs of alternatives. In general, there are $m(m-1)$ ordered pairs of distinct alternatives in X. For a given Arrovian SWF f, if a component Y_i is non–null, then the $m_i(m_i - 1)$ ordered pairs of distinct alternatives from Y_i are not fixed by f. The number of pairs that are fixed by f, then, is

$$\Phi(f) := m(m-1) - \sum [m_i(m_i - 1)], \tag{1}$$

where the sum is taken over all i such that $f|Y_i$ is non-null. We wish to place a lower bound on $\Phi(f)$ within a given class of SWFs.

[5] As pointed out by Campbell and Kelly (2003), Wilson's partition theorem can be easily extend to any domain with the *chain property*. The formal definition of the chain property may be found on p.41 of Campbell and Kelly (2002). Hence, our results also valid for any domain with the chain property.

3 Main Results

The following is immediate from equation (1) but important.

Lemma 2
Suppose that f and f' are Arrovian SWFs which induce the same component structure $\mathcal{Y} = \{Y_1, \cdots Y_k\}$. If $f'|Y_i$ is dictatorial whenever $f|Y_i$ is so, then $\Phi(f') \leq \Phi(f)$.

In particular, given any Arrovian SWF f inducing $\mathcal{Y} = \{Y_1, \cdots, Y_k\}$, let $\underline{f}$ be an Arrovian SWF such that (i) it induces the same component structure $\mathcal{Y}$, and (ii) it is locally dictatorial. Lemma 2 implies that $\Phi(\underline{f}) \leq \Phi(f)$. Thus,

$$\Phi(\underline{f}) = m(m-1) - \sum_{i=1}^{k}[m_i(m_i - 1)]$$

provides the lower bound on the number of fixed pair within a class of Arrovian SWFs inducing the component structure $\mathcal{Y}$. From this reason, we restrict our attention to locally dictatorial Arrovian SWFs from now on.

Given a component structure $\mathcal{Y} = \{Y_1, \cdots, Y_k\}$, let $\{Y_{[1]}, \cdots, Y_{[k]}\}$ be a permutation of components such that $m_{[1]} \geq m_{[2]} \geq \cdots \geq m_{[k]}$. Define the **dominance relation** $\geq_D$ on component structures by

$$\mathcal{Y} \geq_D \mathcal{Y}' \Leftrightarrow \sum_{i=1}^{\ell} m_{[i]} \geq \sum_{i=1}^{\ell} m'_{[i]} \qquad \forall \ell = \{1, \cdots, \min\{k, k'\}\}.$$

Note that this relation is close to the majorization relation defined on real vectors, introduced by Hardy et al. (1934, 1952).[6] The majorization relation compares vectors with the same dimensionality, whereas our dominance relation can compare component structures containing different number of components. However, by adding dummy components which are emptysets to the smaller component structure, these two relations can be consider as equivalent. Namely, suppose that $\mathcal{Y} = \{Y_1, \cdots, Y_k\}$ and $\mathcal{Y}' = \{Y'_1, \cdots, Y'_k\}$ with $k > k'$. Then, $\mathcal{Y} \geq_D \mathcal{Y}'$ iff $(m_1, \cdots, m_k)$ majorizes $(m'_1, \cdots, m'_k, 0, \cdots, 0)$, where zeros implies that $Y'_i = \emptyset$ for $i \in [k'+1, k]$. We say that f **dominates** f' if $\mathcal{Y} \geq_D \mathcal{Y}'$.

We now provide an interpretation of the dominance relation $\geq_D$. For locally dictatorial SWFs, $\geq_D$ compares the inequality of distributions of power between SWFs. That is, "f dominates f'" means that f has an unequal distribution of decisive power compared to f'. To see this point, consider the following example.

Example 1
Suppose that there are three individuals and six alternatives $X = \{x_1, \ldots, x_6\}$. Let f and f' be locally dictatorial SWFs inducing $\mathcal{Y} := \{\{x_1, x_2, x_3\}, \{x_4, x_5\}, \{x_6\}\}$ and $\mathcal{Y}' := \{\{x_1, x_2\}, \{x_3, x_4\}, \{x_5, x_6\}\}$, respectively. Obviously,

$$\mathcal{Y} \geq_D \mathcal{Y}' \text{ and } \neg(\mathcal{Y}' \geq_D \mathcal{Y}).$$

Assume that each individual is a local dictator over exactly one component under both f and f'. Then, under f, the individuals are decisive over 6, 2, and 0 pairs of distinct

[6] For $\alpha, \beta \in \mathbb{R}^k$, α is said to be **majorized** by β if $\sum_{i=1}^{n} \alpha_{[i]} \leq \sum_{i=1}^{n} \beta_{[i]}$, for $n = 1, \cdots, k-1$ and $\sum_{i=1}^{k} \alpha_{[i]} = \sum_{i=1}^{k} \beta_{[i]}$.

alternatives, respectively. On the other hand, under f', each individual is decisive over 2 pair of distinct alternatives. Thus, the decisive power is more unequally distributed under f than under f'. ||

It should be worth mentioning that the component structure $\{X\}$, induced by a globally dictatorial SWF, dominates all other component structures, and the component structure $\{\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}, \{x_5\}, \{x_6\}\}$, induced by a perfectly imposed SWF, is dominated by all other component structures.

We are now ready to offer our generalized trade–off theorem, which state that the number of pairs of alternatives that are socially ranked without consulting anyone's preferences increases as the distribution of dictatorial power becomes equally.

Theorem 1
Suppose that both f and f' are locally dictatorial Arrovian SWFs. If f dominates f', then $\Phi(f) \leq \Phi(f')$ (with equality only if f' dominates f).

Proof:
Let $\mathcal{Y} = \{Y_1, \cdots, Y_k\}$ (resp. $\mathcal{Y}' = \{Y'_1, \cdots, Y'_{k'}\}$) be the component structure induced by f (resp. f'). Let $m_i = |Y_i|$ and $m'_i = |Y'_i|$. Since we assume that f dominates f', we have $k \leq k'$. By adding $k' - k$ dummy elements, we obtain the two vectors $\mathbf{m} = (m_1, m_2, \cdots, m_k, 0, \cdots, 0)$ and $\mathbf{m}' = (m'_1, m'_2, \cdots, m'_{k'})$ in $\mathbb{R}^{k'}$. Note that $\mathbf{m}$ majorizes $\mathbf{m}'$ since we assume that f dominates f'. Define a map $\psi : \mathbb{R}^{k'} \to \mathbb{R}$ by

$$\psi(\mathbf{a}) = \sum_i a_i(a_i - 1).$$

Obviously ψ is symmetric and $\partial\psi(\mathbf{a})/\partial a_i = 2a_i - 1$ is increasing in a_i. Thus, Schur's theorem (Schur, 1923) implies that ψ is Schur–convex, that is, ψ preserves the ordering of majorization (See Theorem A.4 in Marshall and Olkin (1979) p.57). Hence we have $\psi(\mathbf{m}) \geq \psi(\mathbf{m}')$ (with equality only if $\mathbf{m}'$ majorizes $\mathbf{m}$).

Since for any locally dictatorial SWF f, $\Phi(f)$ is given by

$$\Phi(f) = m(m-1) - \psi(\mathbf{m}),$$

then the assertion of the theorem is valid. □

Applying Theorem 1 to Example 1, we obtain $\Phi(f) < \Phi(f')$. In fact, a simple calculation yields that $\Phi(f) = 22$ and $\Phi(f') = 24$. Due to this theorem, we can not only compare the number of fixed pairs, but also provide the lower bound, for a given some class of SWFs, on the number of fixed pairs, by finding the maximal element with respect to the dominance relation.

Thus, as a corollary to Theorem 1 (combined to Lemma 2), we can obtain Campbell and Kelly's trade–off theorem (Campbell and Kelly, 1993, Theorem 2), which provides the lower bound on the number of pairs of fixed alternatives for the class of SWFs such that no individual is a dictator over a subset of alternatives containing at most the fraction t of alternatives.

Let F_δ be the class of all SWFs such that no individual is a dictator on a component of more than δ elements. For any $\delta \in \mathbb{R}$, let $\lfloor \delta \rfloor$ be the greatest integer not larger than δ.

Corollary 1 (Campbell and Kelly's Trade–off Theorem)
Suppose that $0 \leq t \leq 1$ *and* $tm \geq 2$*. Then, for any Arrovian SWF* $f \in F_{tm}$*,*

$$\begin{aligned}\Phi(f) \;\geq\; & m(m-1) - \left\lfloor \frac{m}{\lfloor tm \rfloor} \right\rfloor \cdot \lfloor tm \rfloor \cdot \lfloor tm - 1 \rfloor \\ & - \left(m - \left\lfloor \frac{m}{\lfloor tm \rfloor} \right\rfloor \cdot \lfloor tm \rfloor \right) \cdot \left(m - \left\lfloor \frac{m}{\lfloor tm \rfloor} \right\rfloor \cdot \lfloor tm \rfloor - 1 \right).\end{aligned}$$

Proof:
Suppose that $f \in F_{tm}$ and f induce the component structure $\mathcal{Y}$. Let $\underline{f}$ be a locally dictatorial Arrovian SWF inducing $\mathcal{Y}$. Then, Lemma 2 implies that $\Phi(\underline{f}) \leq \Phi(f)$.

Next, consider any locally dictatorial Arrovian SWF f' inducing the component structure $\mathcal{Y}' = \{Y'_1, \cdots, Y'_k\}$, where $k = \left\lfloor \frac{m}{\lfloor tm \rfloor} \right\rfloor + 1$, such that $m_i = \left\lfloor \frac{m}{\lfloor tm \rfloor} \right\rfloor$ $(i = 1, \cdots, k-1)$ and $m_k = m - \left\lfloor \frac{m}{\lfloor tm \rfloor} \right\rfloor \cdot \lfloor tm \rfloor$. Since $f \in F_{tm}$, each component in $\mathcal{Y}$ contains at most $\left\lfloor \frac{m}{\lfloor tm \rfloor} \right\rfloor$ elements, and hence $\mathcal{Y}' \geq_D \mathcal{Y}$. Then, Theorem 1 implies $\Phi(f') \leq \Phi(\underline{f})$.

Finally, by equation (1), we have

$$\begin{aligned}\Phi(f') \;=\; & m(m-1) - \left\lfloor \frac{m}{\lfloor tm \rfloor} \right\rfloor \cdot \lfloor tm \rfloor \cdot \lfloor tm - 1 \rfloor \\ & - \left(m - \left\lfloor \frac{m}{\lfloor tm \rfloor} \right\rfloor \cdot \lfloor tm \rfloor \right) \cdot \left(m - \left\lfloor \frac{m}{\lfloor tm \rfloor} \right\rfloor \cdot \lfloor tm \rfloor - 1 \right),\end{aligned}$$

which completes the proof. □

4 Applications

4.1 Fixed Numbers of Components

We provide here the lower bound on the number of fixed pairs within a class of SWFs which induce component structures with a fixed number of components. The number of fixed pair is minimized when all but one components are singletons.

Proposition 1
If an Arrovian SWF f *induces the component structure* $\mathcal{Y} = \{Y_1, \cdots, Y_k\}$*, then*

$$\Phi(f) \geq m(m-1) - (m-k+1)(m-k) = (k-1)(2m-k).$$

Proof:
Let $\mathcal{Y}' = \{Y'_1, \cdots, Y'_k\}$ such that $m'_i = 1$ $(i = 1, \cdots, k-1)$ and $m'_k = m - (k-1)$. Then, $\mathcal{Y}' \geq_D \mathcal{Y}$. □

Note that the lower bound of $\Phi(f)$ is linear in m.

4.2 Minimal Participation

We consider here the lower bound on the number of fixed pairs under the constraint which assures minimal participation of individuals, that is, each individual i must have a dictatorial power over a subset of alternatives containing at least m_i alternatives. The number of fixed pairs is minimized when the dictatorial power is concentrated in a single individual.

Proposition 2
Suppose that f is an Arrovian SWF such that each individual i is a dictator on a component of at least m_i elements. Then,

$$\Phi(f) \geq m(m-1) - \sum_{i=2}^{n} m_{[i]}(m_{[i]}-1) - \left(m - \sum_{i=2}^{n} m_{[i]}\right)\left(m - \sum_{i=2}^{n} m_{[i]} - 1\right).$$

Furthermore, $\Phi(f)$ is increasing in $m_{[i]}$ for all $i \geq 2$.

Proof:
Suppose that $\mathcal{Y}$ is induced by f. Let $\mathcal{Y}' = \{Y_1', \cdots, Y_n'\}$ such that $m_1' = m - \sum_{i=2}^{n} m_{[i]}$ and $m_i' = m_{[i]}$ $(i = 2, \cdots, n)$. Then, $\mathcal{Y}' \geq_D \mathcal{Y}$. $\square$

This application is closely related to the well-known *impossibility of a Paretian liberal* shown by Sen (1970, 1976). Sen's liberalism requires that each individual has a pair $\{x, y\}$ of alternatives such that he/she is dictatorial over it. Sen (1970) shows that a Paretian aggregation rule that generates acyclic social preferences violates liberalism. Kelsey (1985) generalizes the result by imposing non-imposition instead of the Pareto principle. In our application, neither the Pareto principle nor non-imposition is not imposed, and thus, we can construct a social welfare function satisfying Sen's liberalism. Note that $m_i = 2$ for all $i \in N$ under liberal social welfare function (or collective choice rule). Proposition 2 implies that

$$\begin{aligned}\Phi(f) &\geq m(m-1) - [(m-2(n-1))(m-2(n-1)-1) + (n-1)2] \\ &= 4(mn - m + n - n^2).\end{aligned}$$

5 Concluding Remarks

This paper investigated social choice without the Pareto principle. Wilson (1972) has shown that a component structure $(\mathcal{Y}, \succ)$ plays a powerful role in the absence of the Pareto principle (Wilson's partition theorem). We introduced a dominance relation on the set of component structures. Based on this dominance relation, we provided a generalized trade–off theorem in the spirit of Campbell and Kelly (1993). As a corollary to our result, we obtained Campbell and Kelly's trade–off theorem (Campbell and Kelly, 1993, Theorem 2). Moreover, we offered two applications. As the first application, we considered the lower bound on the number of fixed pairs when the number of components is fixed. Next, we considered a class of SWFs that there exists a profile $(m_i)_{i \in N}$ such that each individual i is dictatorial on a component with at least m_i elements. We believe that our result can be applicable to further problems.

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